

Math 5710

How can values of a continuous random variable X be generated by using a uniform random number generator?

Suppose X is a continuous random variable and F_X is strictly increasing on an interval $I = (a, b)$, and $F_X(a) = 0$, $F_X(b) = 1$. This means that F_X has an inverse function on the interval (a, b) .

Let U be a uniform random variable on $[0, 1]$ and $W = F_X^{-1}(U)$, a function of the random variable U .

$F_W(x) = P(W \leq x) = P(F_X^{-1}(U) \leq x)$. Applying F_X to both sides of the last inequality, we get $P(F_X^{-1}(U) \leq x) = P(U \leq F_X(x))$. Since U is a uniform random variable, we get $P(U \leq F_X(x)) = F_X(x)$.

We have shown that the cumulative distribution for W is just the cumulative distribution for F_X . Hence $W = X$. To observe values of W , just randomly select values of U and evaluate F_X^{-1} at these values.