

A.

$X \sim N(0, 1)$ or Z

$$f_X(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}, \quad -\infty < z < \infty$$

Find the density function of X^3 .

Let $Y = X^3$.

$$\begin{aligned} 1) \quad F_Y(z) &= P(Y \leq z) = P(X^3 \leq z) \\ &= P(X \leq \sqrt[3]{z}) \\ &= F_X(\sqrt[3]{z}) \end{aligned}$$

$$\begin{aligned} 2) \quad f_Y(z) &= F_Y'(z) = f_X(\sqrt[3]{z}) \cdot \frac{1}{3} z^{-\frac{2}{3}} \\ &= \frac{1}{\sqrt{2\pi}} e^{-\frac{z^{\frac{2}{3}}}{2}} \cdot \frac{1}{3} z^{-\frac{2}{3}}, \quad -\infty < z < \infty \end{aligned}$$

B.

$W \sim \text{exponential}, \lambda = 5$

$$f_W(x) = 5e^{-5x}, \quad x \geq 0$$

Find the density function of W^2

$$\text{Let } T = W^2.$$

$$\begin{aligned} 1) \quad F_T(x) &= P(T \leq x) \\ &= P(W^2 \leq x) \\ &= P(0 \leq W \leq \sqrt{x}) \\ &= F_W(\sqrt{x}) - F_W(0) \\ &= F_W(\sqrt{x}) \end{aligned}$$

$$\begin{aligned} 2) \quad f_T(x) &= F_T'(x) = f_W(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}} \\ &= 5e^{-5\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}, \quad x \geq 0. \end{aligned}$$