Math 5710

If X is a continuous random variable and $Y = g(X)$, how do you find the density function for the random variable Y?

1. Find the cumulative distribution for Y.

$$F_Y(x) = P(Y \leq x) = P[\,g(X) \leq x\,]$$

Now write $P[\,g(X) \leq x\,]$ in terms of just F_X .

example: $X \sim$ N(0,1) , $Y = X^2$

Y is never negative, so for $x \geq 0$

$$F_Y(x) = P(Y \leq x) = P(X^2 \leq x) = P(-\sqrt{x} \leq X \leq \sqrt{x}\ ) =$$

$$F_X(\sqrt{x}\ ) - F_X(-\sqrt{x}\ )$$

2. With $F_Y(x)$ expressed in terms of $F_X(x)$, differentiate to obtain $f_Y(x)$

example continued:

$F_Y(x) = F_X(\sqrt{x}\ ) - F_X(-\sqrt{x}\ )$ Now differentiate both sides:

$$f_Y(x) = f_X(\sqrt{x}\ ) \cdot \frac{1}{2\sqrt{x}} - f_X(-\sqrt{x}\ ) \cdot \frac{-1}{2\sqrt{x}} =$$

$$f_X(\sqrt{x}\ ) \cdot \frac{1}{\sqrt{x}} =$$

$$\frac{1}{\sqrt{x}} \cdot \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x}{2}} \quad \text{for x } \geq 0$$

Note: The density for X^2 is a gamma density with $\lambda = \frac{1}{2}$ and $t = \frac{1}{2}$, more commonly known as a chi-squared distribution with 1 degree of freedom.