

Let Ω be the sample space (set of all possible outcomes) for a given experiment. An event E is a subset of Ω .

What is the probability of E ?

a) Subjective approach: probability as a measure of belief

b) Long-run or relative frequency approach: Repeat an experiment n -times and consider

$$\frac{\text{number of times event } E \text{ occurs}}{n}$$

Now let $P(E) = \lim_{n \rightarrow \infty} \frac{\text{number of times event } E \text{ occurs}}{n}$.

c) Axiomatic approach:

Consider an experiment whose sample space is Ω . For each event A of the sample space Ω we assume that a number $P(A)$ is defined and satisfies the following three axioms.

Axiom 1. $P(\Omega) = 1$

Axiom 2. If $A \subseteq \Omega$, then $P(A) \geq 0$

Axiom 3. If A_1, A_2, A_3, \dots is a sequence of mutually disjoint events, then

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$$