Example:

Consider a circle of radius R and suppose that a point within the circle is randomly chosen in such a manner that all regions within the circle of equal area are equally likely to contain the point. Let the center of the circle be at the origin and define **X** and **Y** to be the coordinates of the point chosen. Find the joint density function of **X** and **Y** . Find the marginal density functions of **X** and **Y .** Let **W** denote the distance from the origin of the point selected. Find the density function for **W**.

Example:

The joint density of **X** and **Y** is given by

$$f(x,y) = \begin{cases} e^{-(x+y)} , & 0 < x < \infty ,\ 0 < y < \infty \\ 0 , & \text{otherwise} \end{cases}$$

Find the density of the random variable **X/Y .**

Example:

Let **X** , **Y** , **Z** be independent and uniformly distributed over the interval (0,1) . Compute P (**Y** $\geq$ **XZ**) .

Suppose **X** and **Y** are independent standard normal random variables. Find the density of **W** = **X** + **Y** .

solution:

Denote the densities for **X** and **Y** by $f_{\mathbf{X}}(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$ and

$f_{\mathbf{Y}}(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}}$. Since **X** and **Y** are independent, their joint density is given by

$f(x,y) = \frac{1}{2\pi} e^{-\frac{x^2}{2}} e^{-\frac{y^2}{2}}$. Now $F_{\mathbf{W}}(w) = P(\mathbf{W} \leq w)$. This probability can be interpreted as integrating f(x,y) over a region in the plane corresponding to the region defined by $\mathbf{W} \leq w$. Hence, $F_{\mathbf{W}}(w) =$

$\int_{-\infty}^{\infty}\int_{-\infty}^{w-x} \frac{1}{2\pi} e^{-\frac{x^2}{2}} e^{-\frac{y^2}{2}} \, dy\, dx$. Then, $f_{\mathbf{W}}(w) = \frac{d}{dw} F_{\mathbf{W}}(w) =$

$\int_{-\infty}^{\infty} \frac{d}{dw} \{\int_{-\infty}^{w-x} \frac{1}{2\pi} e^{-\frac{x^2}{2}} e^{-\frac{y^2}{2}} \, dy\} \, dx = \int_{-\infty}^{\infty} \frac{1}{2\pi} e^{-\frac{x^2}{2}} e^{-\frac{(w-x)^2}{2}} \, dx =$

$\int_{-\infty}^{\infty} \frac{1}{2\pi} e^{-[x^2 - wx + \frac{w^2}{4}] - \frac{w^2}{4}} \, dx = \frac{1}{2\pi} e^{-\frac{w^2}{4}} \int_{-\infty}^{\infty} e^{-[x - \frac{w}{2}]^2} dx$ =(letting $\frac{u}{\sqrt{2}} = x - w$)

$\frac{1}{2\pi\sqrt{2}} e^{-\frac{w^2}{4}} \int_{-\infty}^{\infty} e^{-\frac{u^2}{2}} du = \frac{1}{2\pi\sqrt{2}} e^{-\frac{w^2}{4}} \sqrt{2\pi} = \frac{1}{\sqrt{2\pi}\sqrt{2}} e^{-\frac{w^2}{4}}$.