

Math 5710
Joint Continuous Probability Distributions

Example 1. Suppose X, Y are continuous random variables with joint density

$$f(x, y) = \begin{cases} 2y e^{-x} & \text{for } 0 < y < 1, x > 0 \\ 0 & \text{otherwise} \end{cases}$$

- Find $P(X \leq 3, Y \leq 1)$.
- Find $f_X(x)$ and $f_Y(y)$. Are X, Y independent?
- Find $P(X + Y \leq 1)$. Write your answer as an integral; do not evaluate.

Example 2. The joint probability density function of X and Y is given by

$$f(x, y) = \frac{1}{8}(y^2 - x^2)e^{-y} \quad \text{for } -y \leq x \leq y, 0 < y < \infty$$

Write the density of X in terms of an integral with the appropriate limits of integration. Do not evaluate the integral.

Example 3. The joint probability density function of X and Y is given by

$$f(x, y) = e^{-(x+y)} \quad \text{for } 0 \leq x < \infty, 0 \leq y < \infty$$

Find $P(X < Y)$, $f_X(x)$, and $f_Y(y)$.