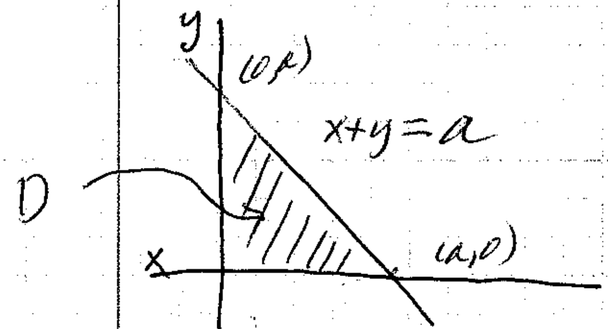
PDF

$X \sim$ Exponential, λ $\qquad f_X(x) = \lambda e^{-\lambda x}, \quad x \geq 0$

$Y \sim$ Exponential, λ $\qquad f_Y(y) = \lambda e^{-\lambda y}, \quad y \geq 0$

independent



$$f(x,y) = \lambda^2 e^{-\lambda(x+y)} = \lambda^2 e^{-\lambda x} e^{-\lambda y}$$

$$W = X + Y \qquad F_W(a) = P[W \leq a] = P[X+Y \leq a]$$

$$= \iint_D \lambda^2 e^{-\lambda x} e^{-\lambda y} \, dx\,dy$$

$$= \int_0^a \int_0^{a-x} \lambda^2 e^{-\lambda x} e^{-\lambda y} \, dy\,dx =$$

$$\int_0^a \lambda e^{-\lambda x} \left[\int_0^{a-x} \lambda e^{-\lambda y} \, dy \right] dx =$$

$$\int_0^a \lambda e^{-\lambda x} \left[-e^{-\lambda y} \Big|_{y=0}^{y=a-x} \right] dx =$$

$$\int_0^a \lambda e^{-\lambda x} \left[1 - e^{-\lambda(a-x)} \right] dx =$$

Note: Fundamental Thm does not apply

$$\int_0^a \left(\lambda e^{-\lambda x} - \lambda e^{-\lambda a + \cancel{\lambda x} - \cancel{\lambda x}}\right) dx =$$

$$\int_0^a \left(\lambda e^{-\lambda x} - \lambda e^{-\lambda a}\right) dx =$$

$$= \left(-e^{-\lambda x} - \lambda e^{-\lambda a} x\right)\Big|_{x=0}^{x=a}$$

$$= -e^{-\lambda a} - \lambda a e^{-\lambda a} - (-1)$$

$$= 1 - e^{-\lambda a} - \lambda a e^{-\lambda a} = 1 - e^{-\lambda a}(1+\lambda a)$$

$$f_W(a) = F_W'(a) = -\lambda e^{-\lambda a} + (1+\lambda a)\lambda e^{-\lambda a}$$

$$= \lambda^2 a e^{-\lambda a}, \quad a \geq 0$$

So $X+Y \sim$ Gamma, $\alpha = 2$, λ