Math 5710

1. Let X be equal to the sum of 400 draws (with replacement) from the box [0, 0, 0, 1]. Find the expected value of X and the standard deviation of X.

2. A certain town has 25,000 families. The average number of children per family is 2.6 with an SD of 0.8. The distribution is not normal, however, since 25% of the families have no children at all. If we draw a random sample of 400 families, what are the chances that between 23% and 27% of the sample families will have no children?

3. Toss a quarter 10 times. Suppose you observed only two heads. Is the coin fair? This result could be just due to chance variation. How likely is it to observe two or fewer heads in ten tosses of a fair coin? Let $X =$ the number of heads in ten tosses of a fair coin. Find $p = P(X \le 2)$. What do you conclude?

4. Consider the following box (population) of 0's and 1's: [? 0's , ? 1's]. The number of 0's in the box is unknown and the number of 1's is also unknown. Let $\mu = \text{Box AV} = \text{proportion of 1's}$. Let $\sigma = \text{Box SD}$.

What is μ? Can we estimate the value of μ?

Let $X_1, X_2, \ldots, X_n$ be a random sample from the box (with replacement). Then

$W_n = \dfrac{X_1 + X_2 + \ldots + X_n}{n}$ is a reasonable estimator for μ. Note that $E(W_n) = \mu$.

Sometimes $W_n < \mu$ and sometimes $W_n > \mu$ but on average $W_n = \mu$. However, W_n is useless as an estimator of μ unless we can determine how accurate it is.

We know that W_n is approximately a normal distribution for n large.

So, $P\left(-2 < \dfrac{W_n - \mu}{\sigma / \sqrt{n}} < 2\right) \approx .95$

Then, $P\left(\dfrac{-2\sigma}{\sqrt{n}} < W_n - \mu < \dfrac{2\sigma}{\sqrt{n}}\right) \approx .95$

and so, $P\left(W_n - \dfrac{2\sigma}{\sqrt{n}} < \mu < W_n + \dfrac{2\sigma}{\sqrt{n}}\right) \approx .95.$

Since $\sigma \le \dfrac{1}{2}$, we have

$P\left(W_n - \dfrac{1}{\sqrt{n}} < \mu < W_n + \dfrac{1}{\sqrt{n}}\right) \approx .95$

Suppose $n = 400$ and $W_n = .65$. Then,

$$\left(.65 - \frac{1}{20}\ ,\ .65 + \frac{1}{20}\right) = (\,.6\ ,\ .7\,)$$

is a 95% confidence interval for μ. We now have obtained a measure of how accurate our estimator W_n is of μ.