Math 5710
Example

An astronomer is interested in measuring, in light years, the distance from his observatory to a distant star. Although the astronomer has a measuring technique, she knows that, because of changing atmospheric conditions and normal error, each time a measurement is made it will not yield the exact distance but merely an estimate. As a result the astronomer plans to make a series of measurements and then use the average value of these measurements as her estimated value of the actual distance. If the astronomer believes that the values of the measurements are independent and identically distributed random variables having a common mean *d* (the actual distance) and a common variance of 4 (light years), how many measurements need she make to be reasonably sure that her estimated distance is accurate to within ½ light year?

X_i = i-th measurement $\quad \bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$

Assume $E(X_i) = d$ light years.

$\text{var } X_i = \sigma^2 = 4$ for all i.

$E(\bar{X}_n) = d$, $\text{var } \bar{X}_n = \frac{1}{n^2} \cdot n \cdot 4 = \frac{4}{n}$

$\sigma_{\bar{X}_n} = \frac{2}{\sqrt{n}}$.

Want $P\left[d - \frac{1}{2} < \bar{X}_n < d + \frac{1}{2}\right] = \boxed{.95}$

$$P\left[-\frac{1}{2} < \bar{X}_n - d < \frac{1}{2}\right] = .95$$

$$P\left[-\frac{1}{2} \cdot \frac{\sqrt{n}}{2} < \frac{\bar{X}_n - d}{2/\sqrt{n}} < \frac{1}{2} \cdot \frac{\sqrt{n}}{2}\right] = .95$$

Central Limit Theorem $\Rightarrow$

$\frac{\sqrt{n}}{4} = 2$, $\sqrt{n} = 8$, $\boxed{n = 64}$