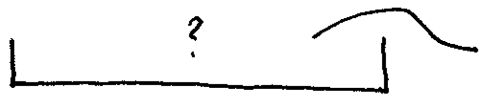
Statistical Sampling



μ, σ^2

$X_1, X_2, \ldots, X_n$

independent

"random sample"

$\forall i, \quad E(X_i) = \mu \quad , \quad \operatorname{var} X_i = \sigma^2$

① How do we estimate μ?

"statistic" $\quad \bar{X} = \dfrac{\sum_{i=1}^{n} X_i}{n} \qquad E(\bar{X}) = \mu$

"unbiased"

Note that

$\operatorname{var} \bar{X} = E(\bar{X} - \mu)^2 = \frac{1}{n^2} \cdot n\sigma^2$

$= \frac{\sigma^2}{n}$

② How do we estimate σ^2?

a) Suppose we know μ. Let

$$W = \frac{1}{n} \sum_{i=1}^{n} (X_i - \mu)^2 .$$

$$E(W) = \frac{1}{n} \cdot n\sigma^2 = \sigma^2$$

"unbiased"

b) What if we do not know μ? Let

$$V = \frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X})^2 \qquad \frac{\sum (X_i - \bar{X})^2}{n-1}$$

$$E(V) = \frac{1}{n} E \sum_{i=1}^{n} \left[(X_i - \mu) - (\bar{X} - \mu) \right]^2$$

$$= \frac{1}{n} E \left\{ \sum_{i=1}^{n} \left[(X_i - \mu)^2 - 2(X_i - \mu)(\bar{X} - \mu) + (\bar{X} - \mu)^2 \right] \right.$$

$$= \frac{1}{n} E \left\{ \sum_{i=1}^{n} (X_i - \mu)^2 - 2(\bar{X} - \mu) \sum_{i=1}^{n} (X_i - \mu) + \sum_{i=1}^{n} (\bar{X} - \mu)^2 \right.$$

$$= \frac{1}{n} E \left\{ \sum_{i=1}^{n} (X_i - \mu)^2 - 2(\bar{X} - \mu) \cdot \left[\sum_{i=1}^{n} X_i - n\mu \right] + n(\bar{X} - \mu)^2 \right\}$$

$$= \frac{1}{n} E \left\{ \sum_{i=1}^{n} (X_i - \mu)^2 - 2(\bar{X} - \mu) \cdot n \left[\frac{\sum X_i}{n} - \mu \right] + n(\bar{X} - \mu)^2 \right\}$$

$$= \frac{1}{n} E \left\{ \sum_{i=1}^{n} (X_i - \mu)^2 - 2(\bar{X} - \mu) \cdot n(\bar{X} - \mu) + n(\bar{X} - \mu)^2 \right.$$

$$= \frac{1}{n} E \left\{ \sum_{i=1}^{n} (X_i - \mu)^2 - n(\bar{X} - \mu)^2 \right\}$$

$$= \frac{1}{n} \left[n\sigma^2 - \sigma^2 \right] = \frac{n-1}{n} \sigma^2$$

biased!!