Chapter 9.2   Homework Answer

Exercises 1-7   Linear Systems   *Solve the system of equations by expressing it in terms of a matrix, and then complete row reduction to achieve echelon form.*

1. $x - 3y = 5$
$3x + y = 5$

Solution:

$\begin{bmatrix} 1 & -3 & 5 \\ 3 & 1 & 5 \end{bmatrix}$ Perform $(-3)R_1 + R_2 \to R_2$. $\begin{bmatrix} 1 & -3 & 5 \\ 1 & 10 & -10 \end{bmatrix}$

From R_2, $10y = -10$, $y = -1$. From R_1, $x - 3y = 5$, $x = 3y + 5 = 3(-1) + 5 = 2$.
Therefore, the solution is given by $x = 2$, $y = -1$.

2. $6x - 12y = 7$
$4x - 8y = -5$

Solution:

Perform $2R_1 \to R_1$, $(-3)R_2 \to R_2$, and $R_1 + R_2 \to R_2$: $\begin{bmatrix} 12 & -24 & 14 \\ 0 & 0 & 29 \end{bmatrix}$. From R_2, $0 \cdot x + 0 \cdot y = 29$.
There are no numbers x, y that will satisfy this equation. The system is inconsistent.

3. $x + 2y + z = 3$
$-3x + 4z = 5$
$-3y + 2z = 1$

Answer:

$x = \frac{3}{11}, y = \frac{7}{11}, z = \frac{16}{11}$

Solution:

In matrix form the given system becomes $\begin{bmatrix} 1 & 2 & 1 & 3 \\ -3 & 0 & 4 & 5 \\ 0 & -3 & 2 & 1 \end{bmatrix}$. Eliminate x in R_2.

$$R_2 + 3R_1 \to R_2 \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 6 & 7 & 14 \\ 0 & -3 & 2 & 1 \end{bmatrix} \xrightarrow[2R_2+R_3 \to R_3]{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & -3 & 2 & 1 \\ 0 & 0 & 11 & 16 \end{bmatrix}$$

Now by back-substitution we get $z = 16/11$, $y = 7/11$, and $x = 3/11$.

4. $x + 2y + z = 1$
$-2x + y - 2z = -2$
$-x + 8y - z = 2$

Solution:

In matrix form the given system becomes $\begin{bmatrix} 1 & 2 & 1 & 1 \\ -2 & 1 & -2 & -2 \\ -1 & 8 & -1 & 2 \end{bmatrix}$. $2R_1 + R_2 \to R_2 R_1 + R_3 \to R_3$

$\begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & 5 & 0 & 0 \\ 0 & 10 & 0 & 3 \end{bmatrix}$. From R_2, $5y = 0$, and from R_3, $10y = 3$. We get no solution since there is no number y that satisfies $5y = 0$ and $10y = 3$. The system is inconsistent.

5.

$$\begin{aligned} x + y + 3z &= -1 \\ 3x - 4z &= -4 \\ -x + 2y + 2z &= 2 \end{aligned}$$

Answer:

$x = -\frac{4}{3},\ y = \frac{1}{3},\ z = 0$

Solution:

In matrix form the given system becomes $\begin{bmatrix} 1 & 1 & 3 & -1 \\ 3 & 0 & -4 & -4 \\ -1 & 2 & 2 & 2 \end{bmatrix}$. Eliminate x from R_2 and R_3.

$\xrightarrow[R_1+R_3\to R_3]{R_2-3R_1\to R_2} \begin{bmatrix} 1 & 1 & 3 & -1 \\ 0 & -3 & -13 & -1 \\ 0 & 3 & 5 & 1 \end{bmatrix} \xrightarrow{R_2+R_3\to R_3} \begin{bmatrix} 1 & 1 & 3 & -1 \\ 0 & -3 & -13 & -1 \\ 0 & 0 & -8 & 0 \end{bmatrix}$. Now by back-substitution we get $z = 0$, $y = 1/3$, and $x = -4/3$.

6.

$$\begin{aligned} 3x + 4y - 4z &= -1 \\ 6x - 2y - 2z &= -2 \\ y - 3z &= -3 \end{aligned}$$

Solution:

In matrix form the given system becomes $\begin{bmatrix} 3 & 4 & -4 & 1 \\ 6 & -2 & -2 & -2 \\ 0 & 1 & -3 & -3 \end{bmatrix}$. Eliminate x in R_2.

$\xrightarrow{R_2-2R_1\to R_2} \begin{bmatrix} 3 & 4 & -4 & 1 \\ 0 & -10 & 6 & 0 \\ 0 & 1 & -3 & -3 \end{bmatrix} \xrightarrow[10R_2+R_3\to R_3]{R_2\leftrightarrow R_3} \begin{bmatrix} 4 & 4 & -4 & 1 \\ 0 & 1 & -3 & -3 \\ 0 & 0 & -24 & -30 \end{bmatrix}$.

From R_3, $-24z = -30$, $z = 30/24 = 5/4$. From R_2, $y = 3z - 3 = 3(5/4) - 3 = 3/4$.
Then from R_1, $3x + 4y - 4z = -1$, $x = (-3 + 5 - 1)/3 = 1/3$.

7.

$$\begin{aligned} 2x - y - 3z &= 1 \\ x + y + 5z &= 2 \\ 3x + 2z &= 3 \end{aligned}$$

Solution:

First perform $R_2 \leftrightarrow R_1$ to get $\begin{bmatrix} 1 & 1 & 5 & 2 \\ 2 & -1 & -3 & 1 \\ 3 & 0 & 2 & 3 \end{bmatrix} \xrightarrow[-3R_1+R_3 \to R_3]{-2R_1+R_1 \to R_2} \begin{bmatrix} 1 & 1 & 5 & 2 \\ 0 & -3 & -13 & -3 \\ 0 & -3 & -13 & -3 \end{bmatrix}$

$-R_2 + R_3 \to R_3 \quad \begin{bmatrix} 1 & 1 & 5 & 2 \\ 0 & -3 & -13 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. R_3 represents the equation $0x + 0y + 0z = 0$. Therefore the system is dependent and has infinitely many solutions. If $z = k$, then $y = (-13k + 3)/3$, and from R_1, $x = (3 - 2k)/3$.