# Three–Dimensional Statistical Graphics Based on Interactively Animated Anaglyphs

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## Contents

# 1 Introduction

## Anaglyphs

- a kind of stereoscopic graphics
- two projections of a three-dimensional figure
- usually drawn in complementary colors red and green
- looked at through filter glasses of the same colors
- onlooker is expected to obtain the impression of a three-dimensional object

# 2 Viewing Anaglyphs

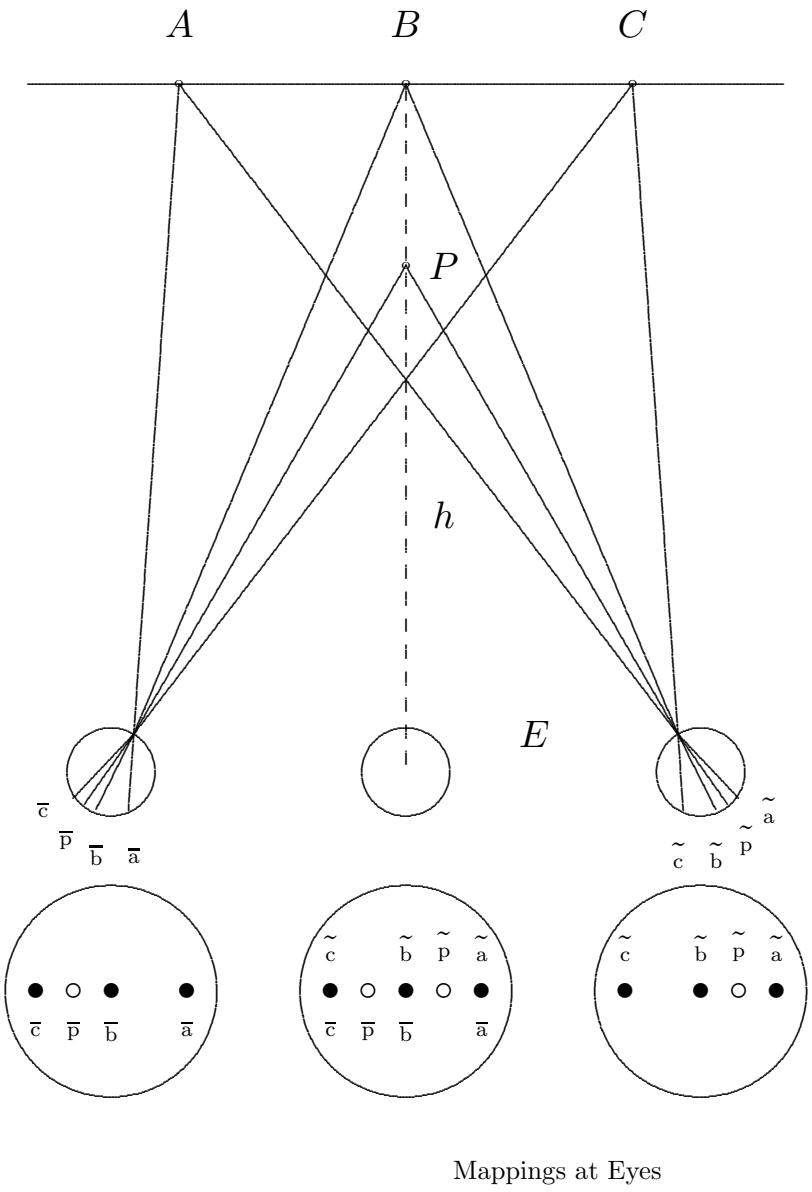


Figure 1: Natural Viewing

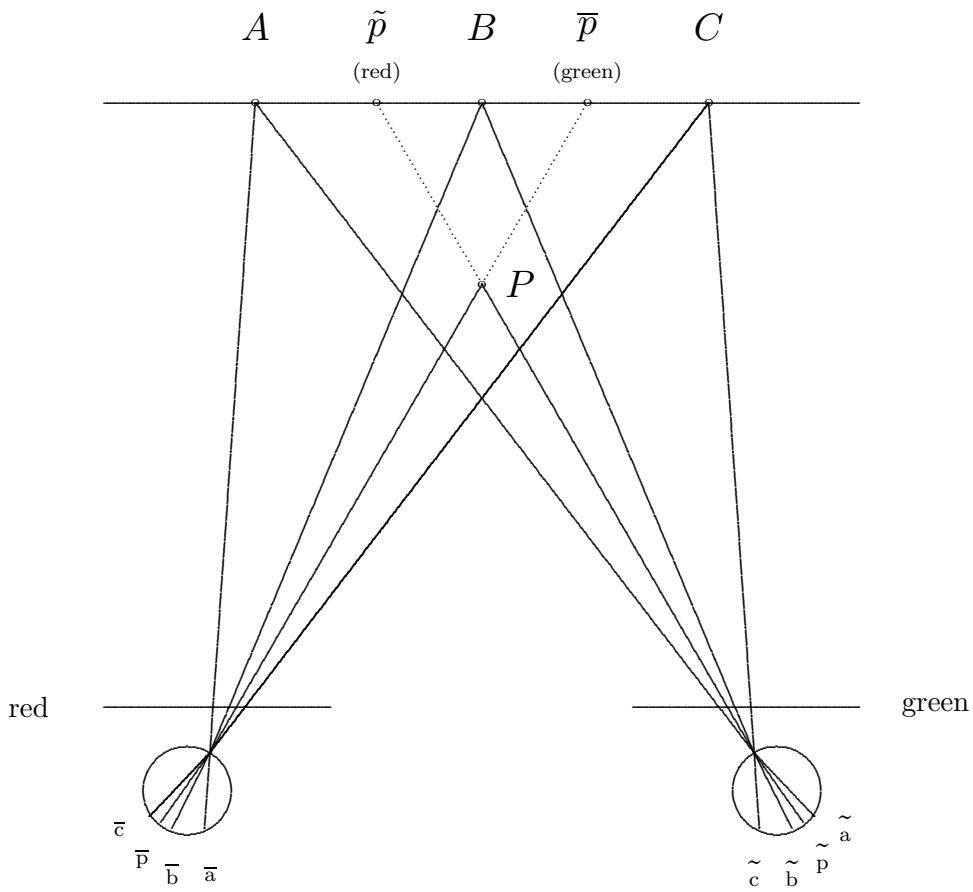


Figure 2: Viewing Anaglyphs

# 3 History and Examples

- German teacher Wilhelm Rollmann described the effect of stereoscopic graphics drawn in red and green colors and looked at with the naked eye (Rollmann (1853a))
- Rollmann (1853b) described the effect of looking at such colored pictures using filter glasses of corresponding complementary colors => birth of anaglyphs
- French teacher Joseph Charles d'Almeida (1858): used differently colored light to produce anaglyphs
- name „anaglyps“ introduced by the French Ducos du Hauron in 1891
- 1912: one of the first books dealing with anaglyphs (Vuibert (1912))
- 1930/40's: anaglyphs used in geometry (Köhler, Graf & Calov (1938), Graf (1938, 1941))
- 1930/40's: anaglyphs used for mining and related applications (Rellensmann/Jung (1939), Rellensmann (1940), and Linhard (1940))
- Hungarian Pál (1961, 1974) used anaglyphs for geometry, mechanical engineering, architecture, chemistry, and spatial mathematical problems

- 1960's: anaglyphs used in chemistry (Hollemann/Wiberg (1963) and Klages (1965))

- Burkhardt (1963, 1972, 1974) covers technical aspects and problems of printed anaglyphs such as optimal colors, best filter glasses, etc.

- other applications: architecture, cartography, biology, medicine, and CAD applications

- statistics: ???

- Banchoff (1986), Carr, Littlefield & Hall (1986), and Gabriel/Odoroff (1986), all published in Wegman/DePriest (1986)

- Huber (1987):

  "Statisticians still lag behind other scientists in their use of stereo pairs. An easily accessible example in molecular biology is the "Stereo Supplement" to the text book by Dickerson and Geis (1969). Almost every other issue of *Science* has articles with stereo illustrations; for a particulary nice enhanced dot–plot (in colors) see Bash et al. (1983). The statisticians' efforts (e. g., the plates in Wegman and DePriest 1986) come late and pale in comparison."

# 4 Anaglyphs and Statistics

- since 1988 research on anaglyphs within statistics: Statistical Department, University of Dortmund, Germany by Hering, von der Weydt and Symanzik (Hering/von der Weydt (1989), Hering/Symanzik (1992) and Symanzik (1991, 1993))

- main purpose: development of computer software to interactively animate anaglyphs for multivariate statistical applications

- real–time three–dimensional animation considerably increases the three–dimensional effect of anaglyphs

- every application created for interactive systems for graphical data analysis since Tukey's PRIM-9 in 1972 can be done with anaglyph programs

**Applications within Statistics**

- three–dimensional scatterplots or biplots instead of bivariate scatterplots
- mixed mappings
- cartographic mappings
- spatial statistics
- time series
- three–dimensional histograms
- three–dimensional growth curves
- bivariate probability/cumulative density functions or any bivariate function
- contour plots

# 5 Mathematical Calculus

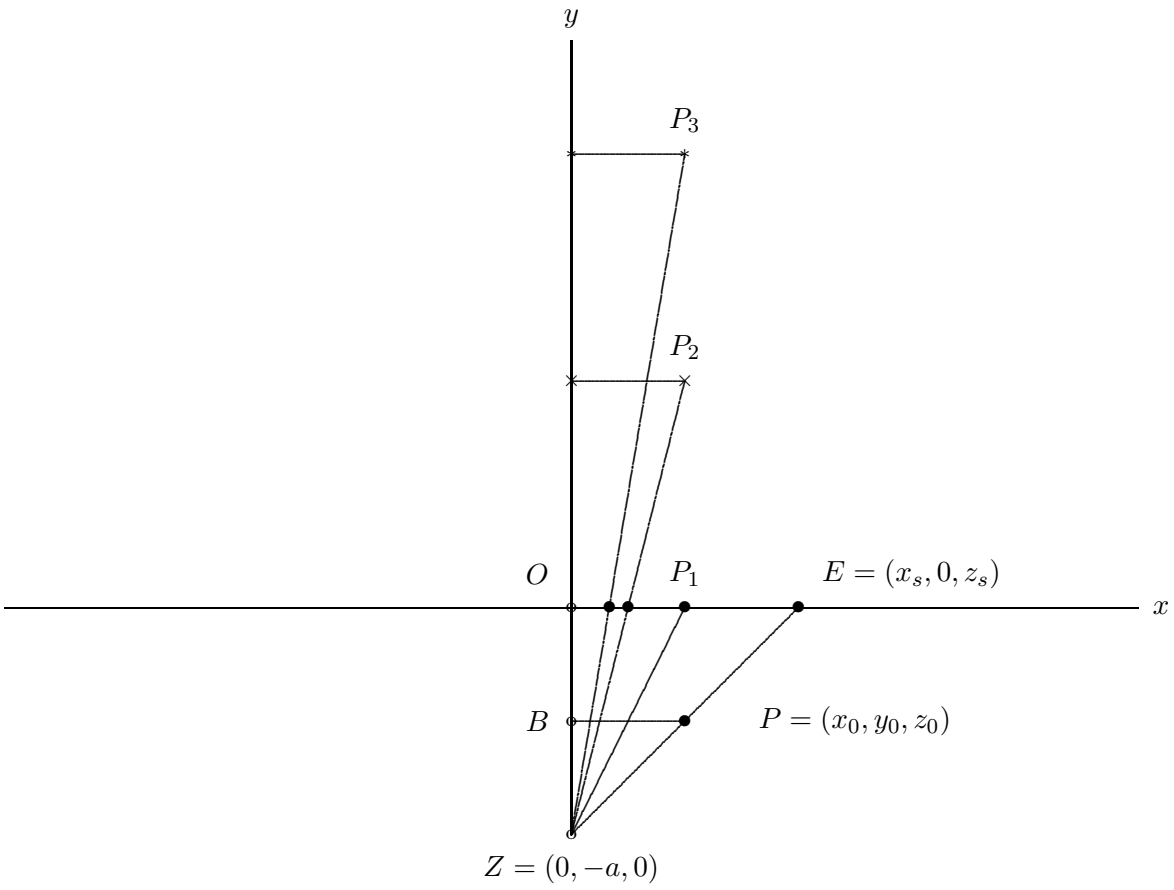


Figure 3: Projection into the $xy$ Plane for a Single COP

CRT coordinates $(x_p, y_p)$

$$\frac{\mid PB \mid}{\mid EO \mid} = \frac{\mid ZB \mid}{\mid ZO \mid} \Leftrightarrow \mid EO \mid = \mid PB \mid \cdot \frac{\mid ZO \mid}{\mid ZB \mid}$$

Therefore

$$x_p = x_s = x_0 \cdot \frac{a}{y_0 + a}.$$

By analogy (as projection into the $zy$ plane)

$$y_p = z_s = z_0 \cdot \frac{a}{y_0 + a}$$

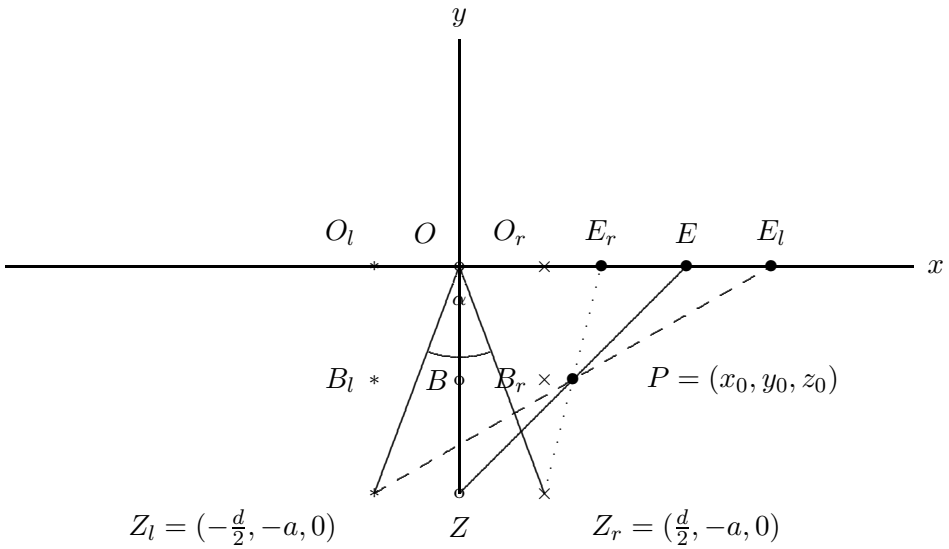


Figure 4: Projection into the $xy$ Plane for Two COPs

Here

$$\mid E_l O_l \mid = \mid PB_l \mid \cdot \frac{\mid Z_l O_l \mid}{\mid Z_l B_l \mid} = \left| \left( x_0 + \frac{d}{2} \right) \cdot \frac{a}{y_0 + a} \right|$$

and

$$\mid E_r O_r \mid = \mid PB_r \mid \cdot \frac{\mid Z_r O_r \mid}{\mid Z_r B_r \mid} = \left| \left( x_0 - \frac{d}{2} \right) \cdot \frac{a}{y_0 + a} \right|$$

Therefore

$$x_{l_p} = \left( x_0 + \frac{d}{2} \right) \cdot \frac{a}{y_0 + a} - \frac{d}{2},$$
$$x_{r_p} = \left( x_0 - \frac{d}{2} \right) \cdot \frac{a}{y_0 + a} + \frac{d}{2},$$

and

$$y_{l_p} = y_{r_p} = y_p = z_0 \cdot \frac{a}{y_0 + a}.$$

# 6 Comparision of Different Stereoscopic CRT Display Systems

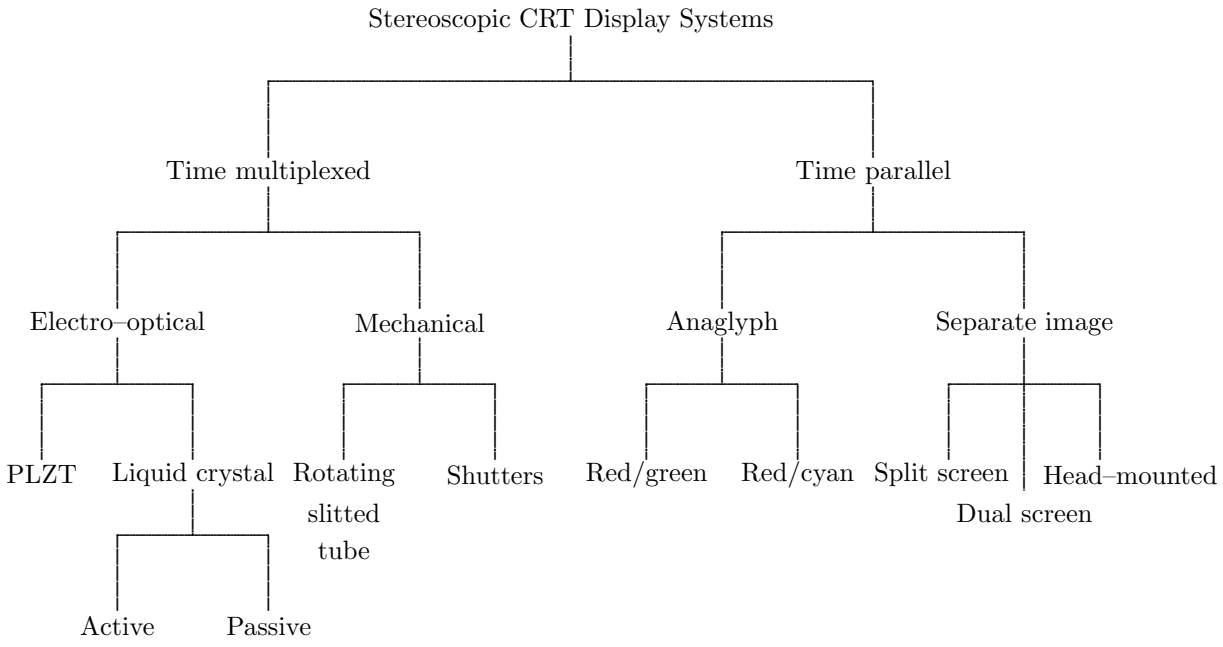


Figure 5: Types of Stereoscopic CRT Display Systems (Hodges (1992), Figure 4)

**Main advantage of anaglyphs:**

- easy to construct
- available for CRT's and for paper
- only cheap additional „hardware“, i. e. filter glasses, required