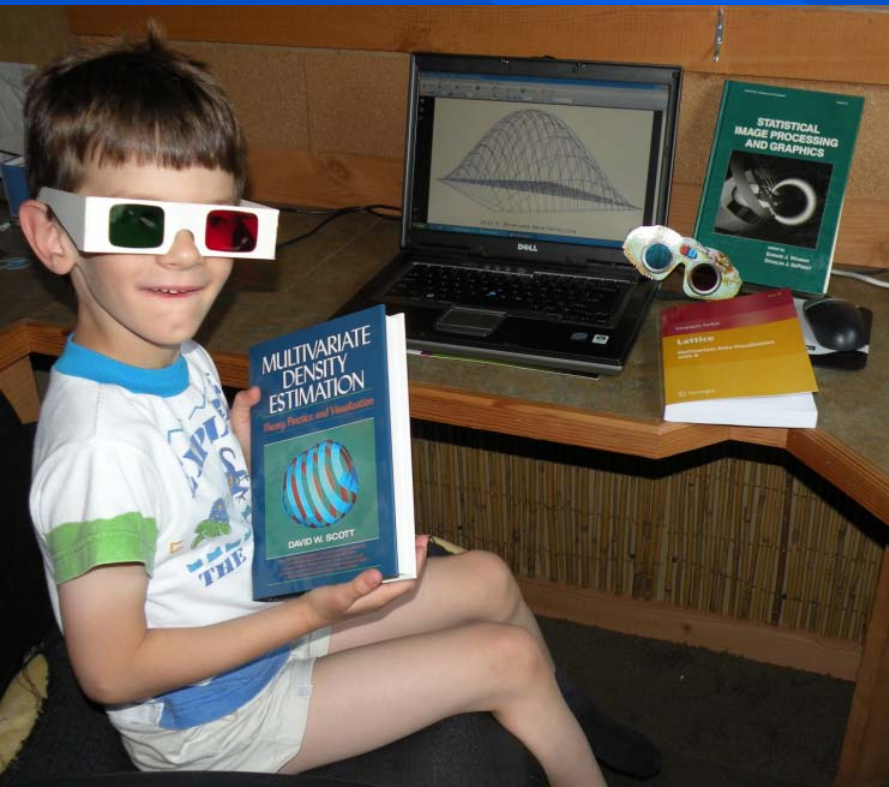


3-D Stereoscopic Plots: From History to R

Jürgen Symanzik

Utah State University, Logan, UT, USA

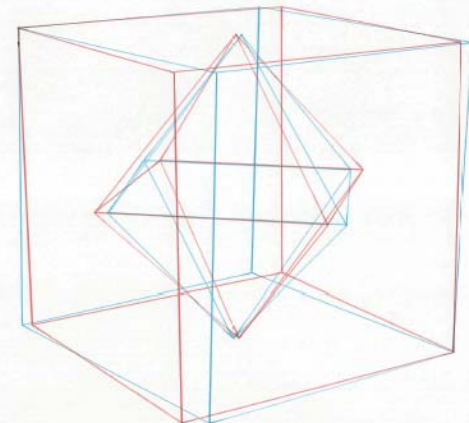
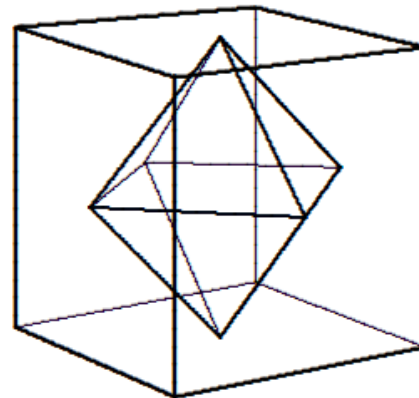
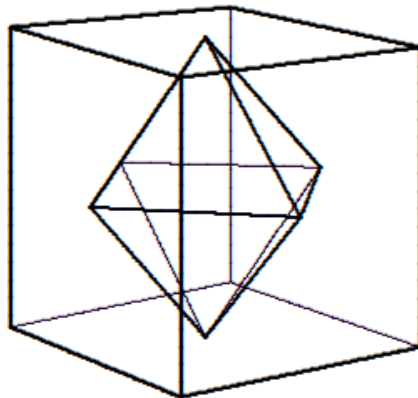


e-mail: symanzik@math.usu.edu

<http://www.math.usu.edu/~symanzik>

Contents

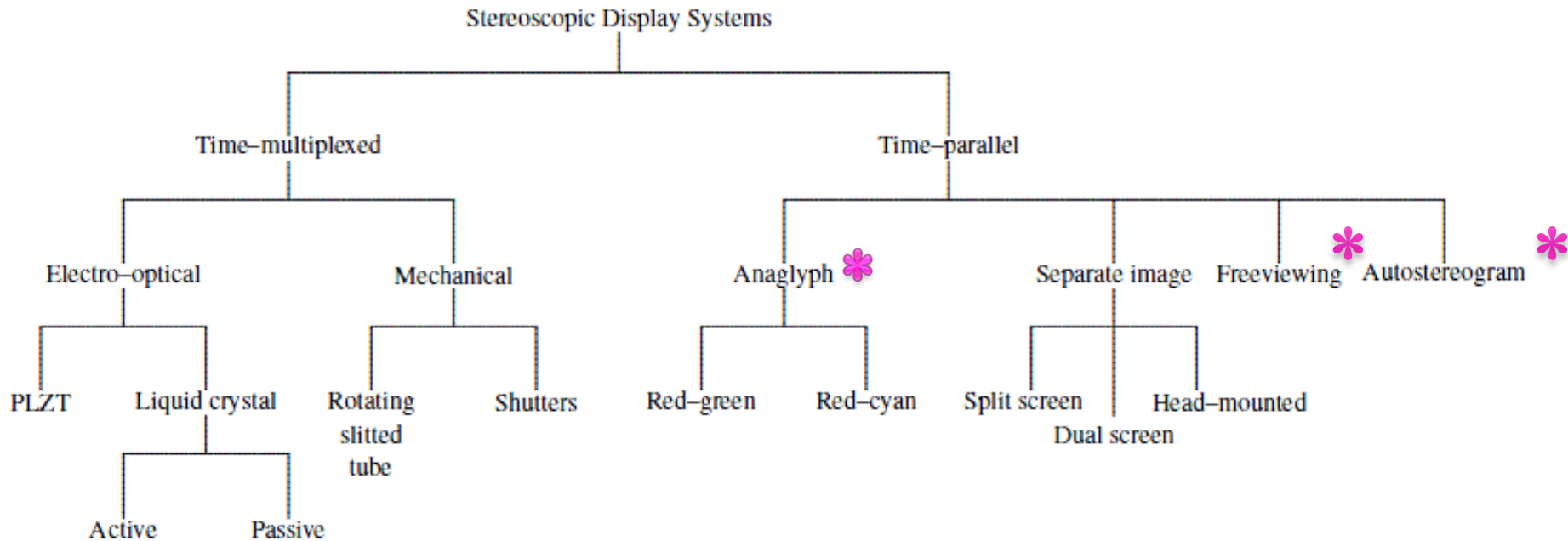
- Background on 3-D Stereoscopic Graphics
- History
- Mathematical Calculations
- Applications in Statistics: Then ...
- Applications in Statistics: ... & Now
- Outlook



Background

- 3-D Stereoscopic Graphics allow human viewers to interpret plots as realistic 3-D images
- Exists for paper, computer screen, or projections
- 3-D effect because each eye sees a slightly different image
- Human brain combines both images to 3-D image

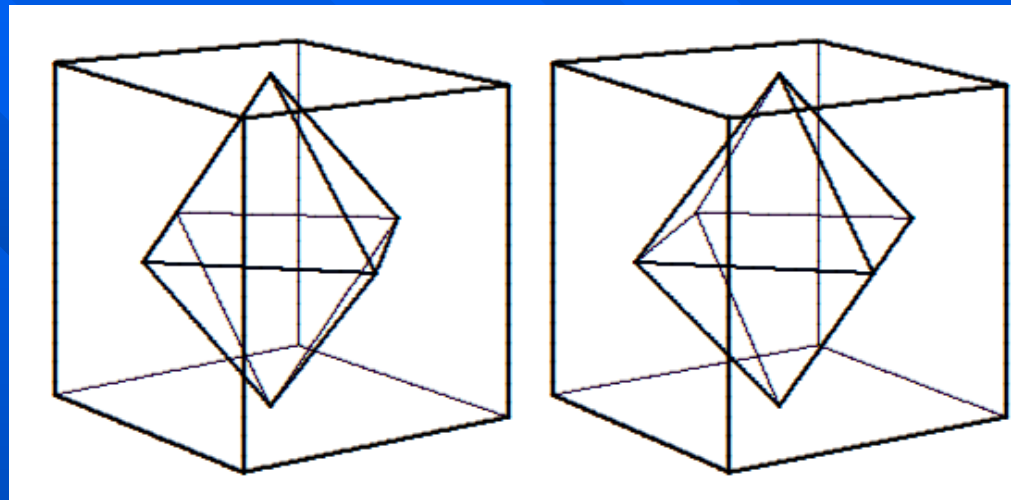
Types of Stereoscopic Displays



Adapted and extended from Hodges (1992); * discussed here.

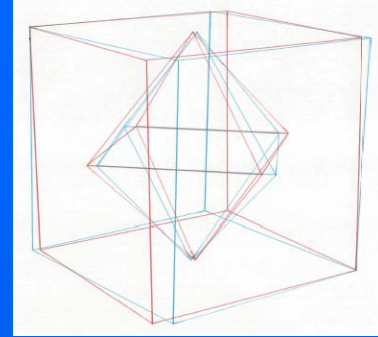
Freeviewing of Side-by-Side Images

- Viewer looks at 2 slightly different images, drawn side-by-side



- No tools needed
- Viewing techniques
 - Focus on point before viewing plane
 - Focus on point behind viewing plane
- One 3-D image & 2 ghost images

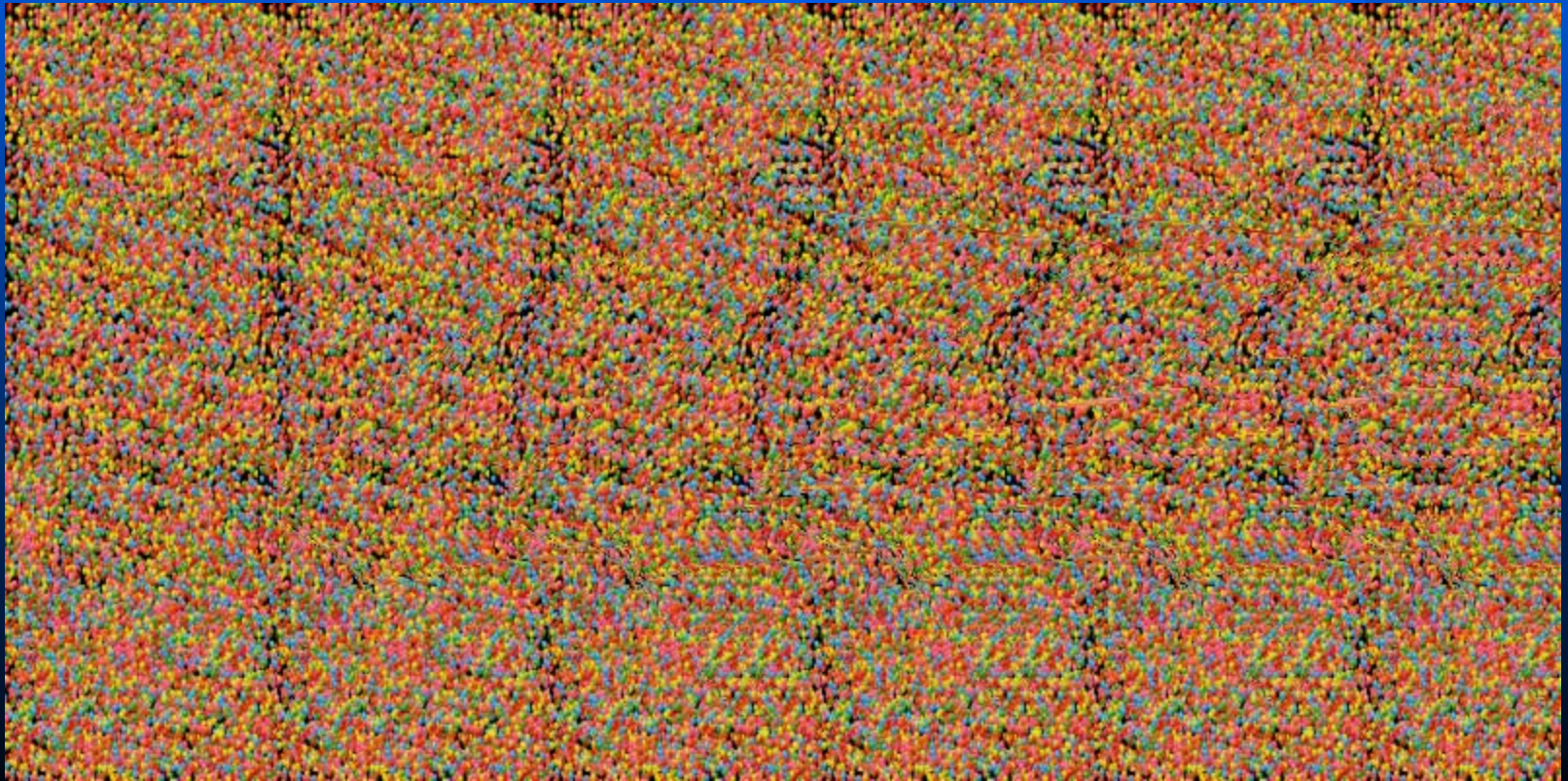
Anaglyphs



- Word “anaglyphos” originates from Greek
- Meaning: object is shaped relief-like, i.e., has a non-flat surface
- Two projections of a 3-D image looked at through filter glasses
- Common colors: red-green (print), red-cyan (CRT), also: red-blue & magenta-green
- Filter glasses filter out one of the images
- Brain combines different images into 3-D image

Autostereograms (1)

- Look like random noise or colorful patterns
- 3-D image revealed when looked at in a technique similar to the freeviewing of side-by-side images



Autostereograms (2)

File:Stereogram Tut Random Dot Shark.png - Wikipedia, the free encyclopedia - Windows Internet Explorer

http://en.wikipedia.org/wiki/File:Stereogram_Tut_Random_Dot_Shark.png

animated autostereogram

File Edit View Favorites Tools Help

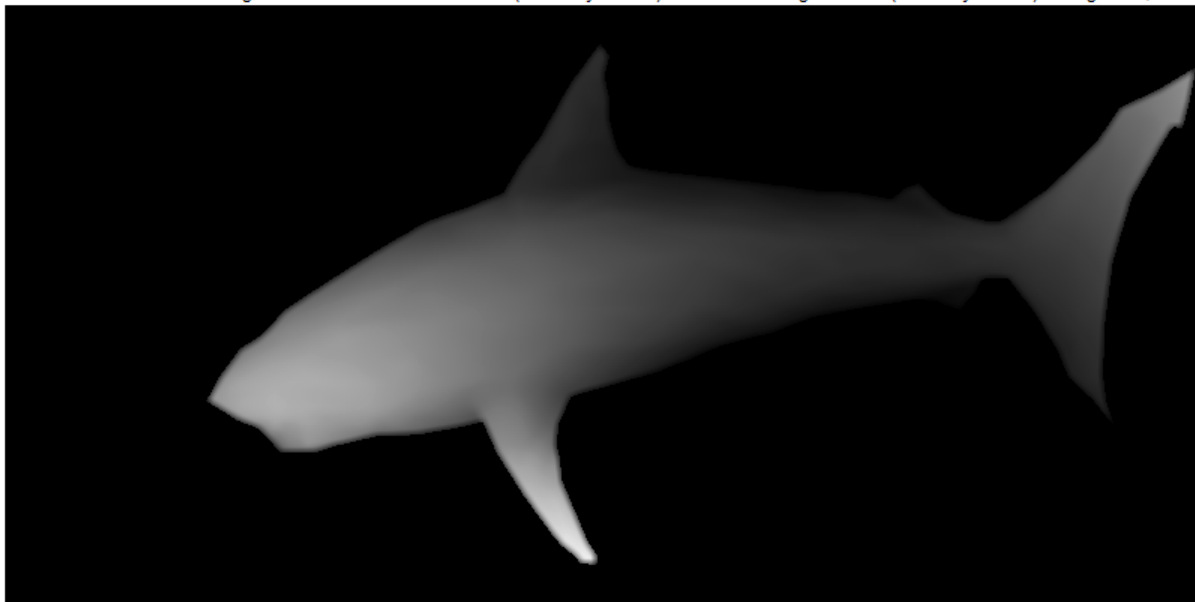
Google autostereogram Search Translate More >> Sign In Convert Select

Favorites File:Stereogram Tut Random Dot Shark.png - Wikipedi...

Autostereogram Tutorial Random Dot Shark

- Description: [Random dot stereogram](#) showing a shark, to be watched with "wall-eyed"
- Source: Fred Hsu, March 2005.

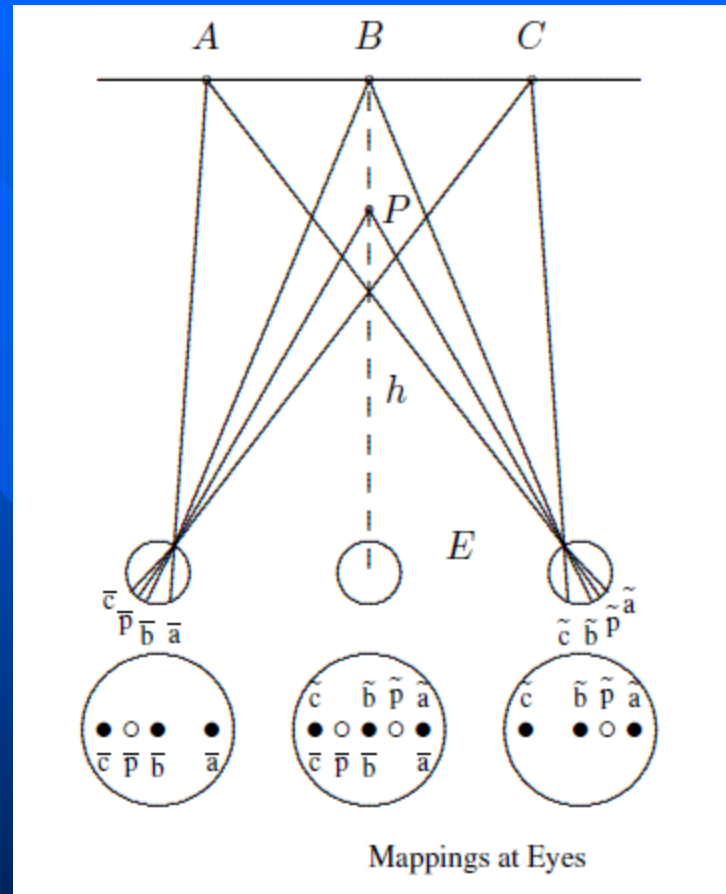
This random dot autostereogram encodes a 3D scene of a (colorfully dotted) shark swimming before a (colorfully dotted) background, like this:



It can be "seen" with proper "wall-eyed" [viewing technique](#): the eyes have to be oriented as if focusing on an object behind the image, causing two neighboring repetition of the dot pattern to merge and the shark to appear.

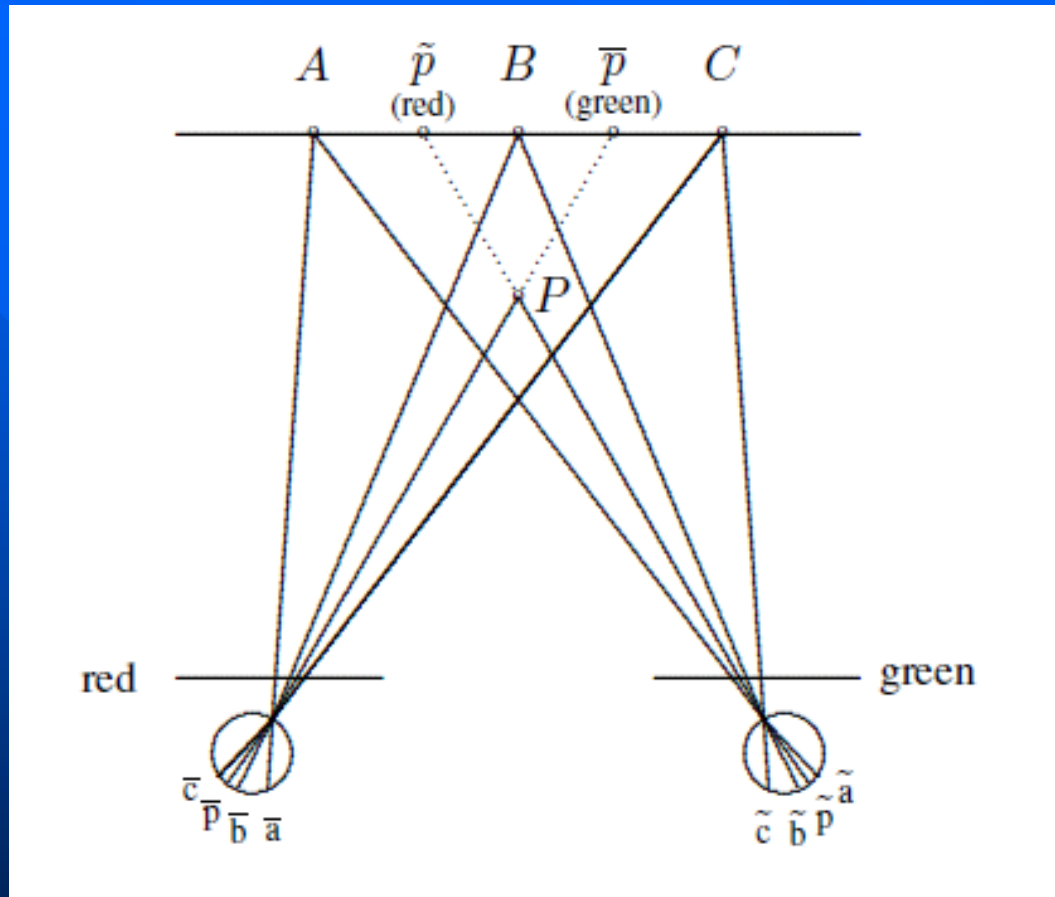
I, the copyright holder of this work, hereby publish it under the following licenses:

Natural Viewing



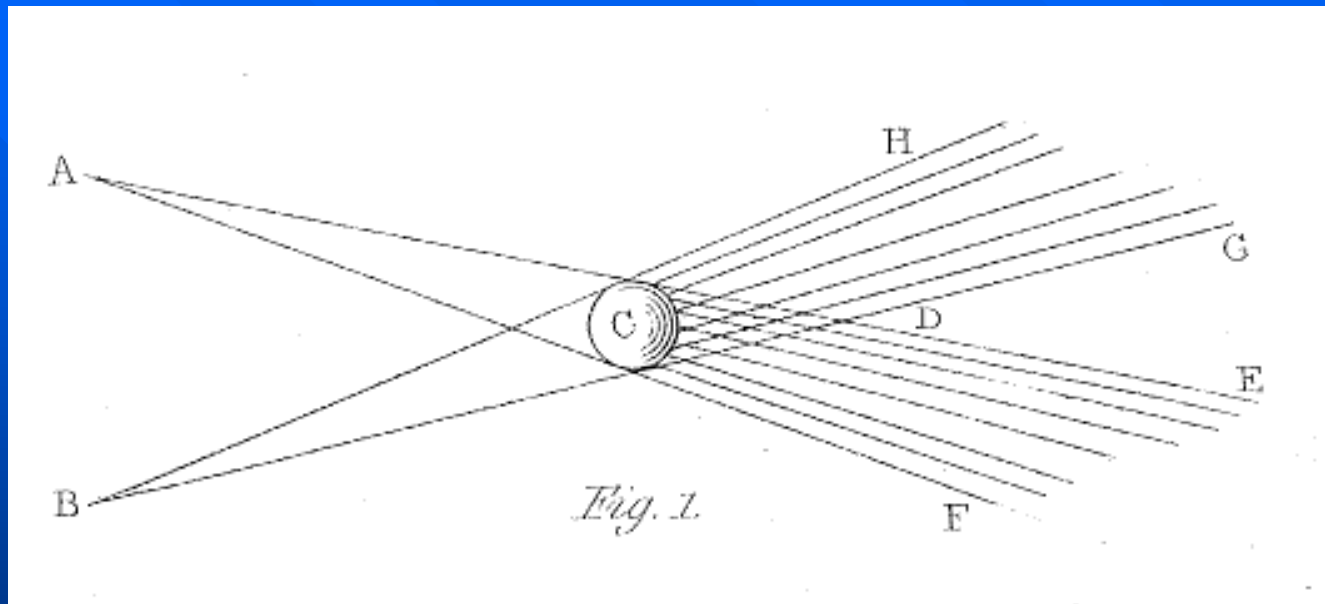
- Natural viewing of a point P that is located in front of a plane, compared to points A , B , and C that are located in a plane

3-D Viewing via Anaglyphs



- Viewing experience for anaglyphs where red and green filter glasses filter out what becomes invisible for each eye

History: Leonardo da Vinci's (1452-1519) Failure ...



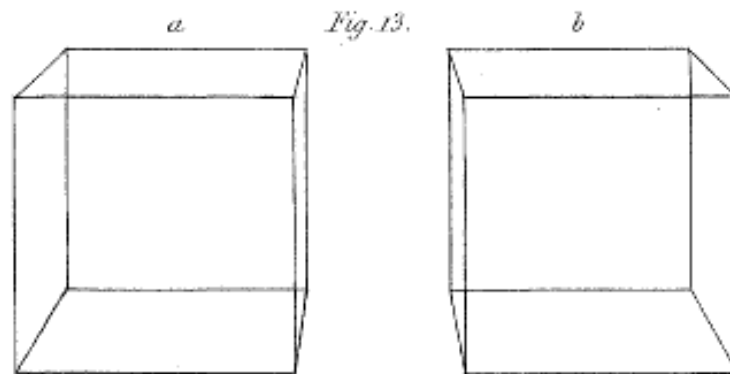
■ Wheatstone (1838) explains:

Had LEONARDO DA VINCI taken, instead of a sphere, a less simple figure for the purpose of his illustration, a cube for instance, he would not only have observed that the object obscured from each eye a different part of the more distant field of view, but the fact would also perhaps have forced itself upon his attention, that the object itself presented a different appearance to each eye. He failed to do this, and no subsequent writer within my knowledge has supplied the omission; the projection of two

... is Wheatstone's Success (1838)

XVIII. *Contributions to the Physiology of Vision.—Part the First. On some remarkable, and hitherto unobserved, Phenomena of Binocular Vision.* By CHARLES WHEATSTONE, F.R.S., Professor of Experimental Philosophy in King's College, London.

Received and Read June 21, 1838.



each eye successively while the other is closed. Plate XI. fig. 13. represents the two perspective projections of a cube; *b* is that seen by the right eye, and *a* that presented to the left eye; the figure being supposed to be placed about seven inches immediately before the spectator.

Rollmann's (1853) Description of Anaglyphs

XII. Zwei neue stereoskopische Methoden; von W. Rollmann.

Zu der großen Zahl der schon bekannten, durch Wheatstone, Dove, Brewster und Wilde entdeckten, stereoskopischen Methoden habe ich folgende neue gefunden.

2. Farbenstereoskop, bestehend aus einer farbigen Doppelzeichnung und zwei gefärbten Gläsern.

Man zeichnet zwei zusammengehörige Körperansichten um *denselben* Mittelpunkt, die eine für das rechte, die andere für das linke Auge. Wenn es nun ein Mittel giebt,

More History on Anaglyphs (1)

- French teacher Joseph Charles d'Almeida (1858): used differently colored light to produce anaglyphs
- Name “anaglyphs” introduced by the French Ducos du Hauron in 1891
- 1912: one of the first books dealing with anaglyphs (Vuibert, 1912)
- 1930/40ies: anaglyphs used in geometry (Koehler, Graf & Calov, 1938; Graf, 1938,1941)

More History on Anaglyphs (2)

- Burkhardt (1963, 1972, 1974) covered technical aspects and problems of printed anaglyphs such as optimal colors, best filter glasses, etc.
- Ideses and Yaroslavsky (2004, 2005) presented new methods how to improve the visual appearance of anaglyphs

I Ideses and L. Yaroslavsky



Figure 5. Standard colour anaglyph without image registration (top image) and anaglyph enhanced by means of image registration (bottom image). Note the reduced ghosting artefacts in the enhanced anaglyph. Use the red filter for the left eye and the blue filter for the right eye.

Mathematical Calculations (1)

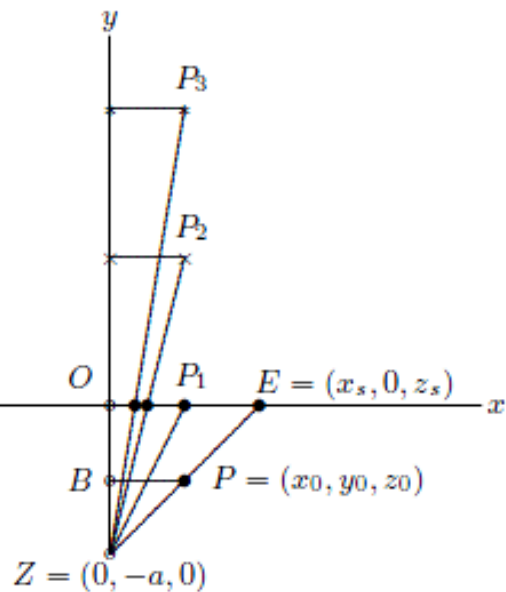


Figure 9: Projection for a single COP.

CRT coordinates (x_p, y_p)

$$\frac{|PB|}{|EO|} = \frac{|ZB|}{|ZO|} \Leftrightarrow |EO| = |PB| \cdot \frac{|ZO|}{|ZB|}$$

Therefore

$$x_p = x_s = x_0 \cdot \frac{a}{y_0 + a}$$

By analogy (as projection into the zy plane)

$$y_p = z_s = z_0 \cdot \frac{a}{y_0 + a}$$

Mathematical Calculations (2)

Here

$$|E_l O_l| = |PB_l| \cdot \frac{|Z_l O_l|}{|Z_l B_l|} = \left| \left(x_0 + \frac{d}{2} \right) \cdot \frac{a}{y_0 + a} \right|$$

and

$$|E_r O_r| = |PB_r| \cdot \frac{|Z_r O_r|}{|Z_r B_r|} = \left| \left(x_0 - \frac{d}{2} \right) \cdot \frac{a}{y_0 + a} \right|$$

Therefore

$$x_{l_p} = \left(x_0 + \frac{d}{2} \right) \cdot \frac{a}{y_0 + a} - \frac{d}{2},$$

$$x_{r_p} = \left(x_0 - \frac{d}{2} \right) \cdot \frac{a}{y_0 + a} + \frac{d}{2},$$

$$y_{l_p} = y_{r_p} = y_p = z_0 \cdot \frac{a}{y_0 + a}.$$

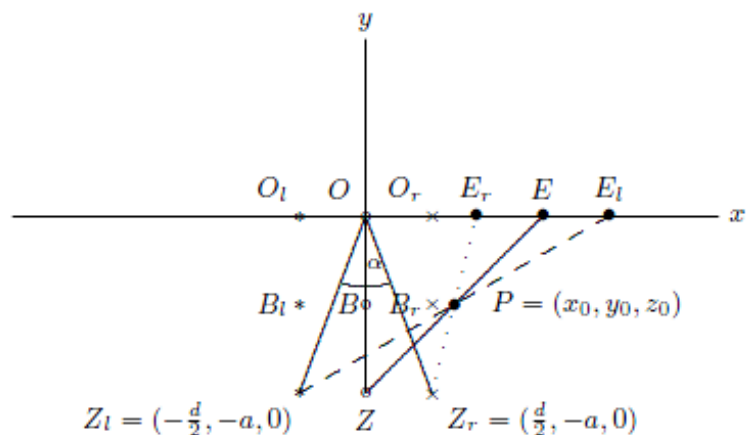


Figure 10: Projection for two COPs. Subscripts l and $*$ relate to the left eye, subscripts r and \times relate to the right eye, and no subscript and o relate to the imaginary central eye.

Applications in Statistics: Then ... (1)

- Freeviewing of side-by-side images:
Huber (1987)

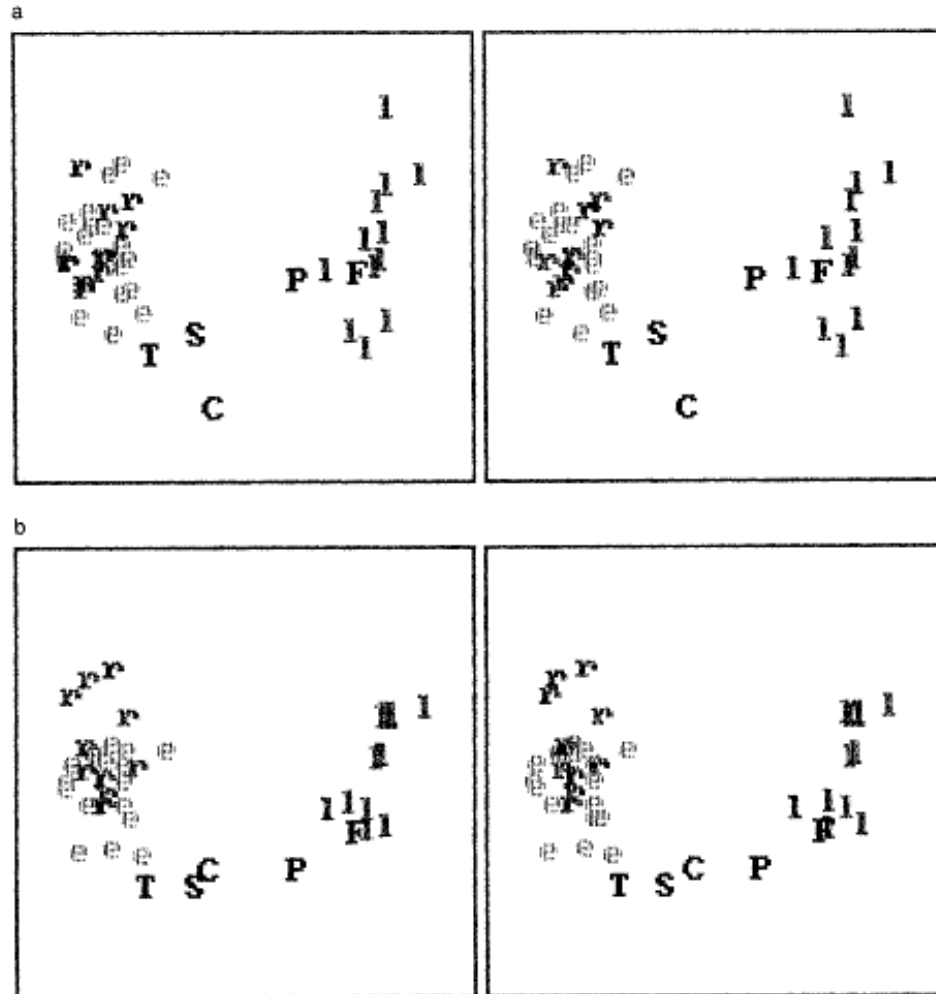


Figure 1. Results of a Principal Component Analysis of the Plato Sentence-Ending Data: First Three Principal Scores (a) Viewed Along the Third Principal Axis and (b) Rotated by 45° Around the First Axis. r—the books of the Republic; e—the other early works; l—the laws; T—Timaeus; S—Sophist; C—Critias; P—Politicus; F—Philebus.

Applications in Statistics: Then ... (2)

- Freeviewing of side-by-side images:

Henderson (1989)

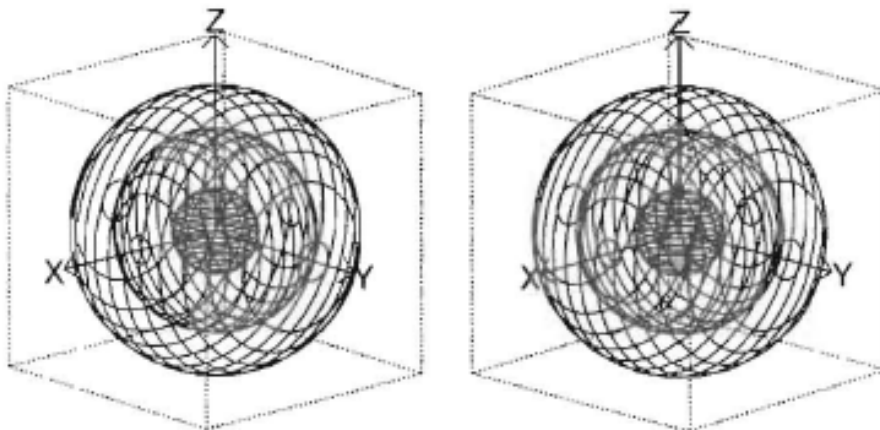
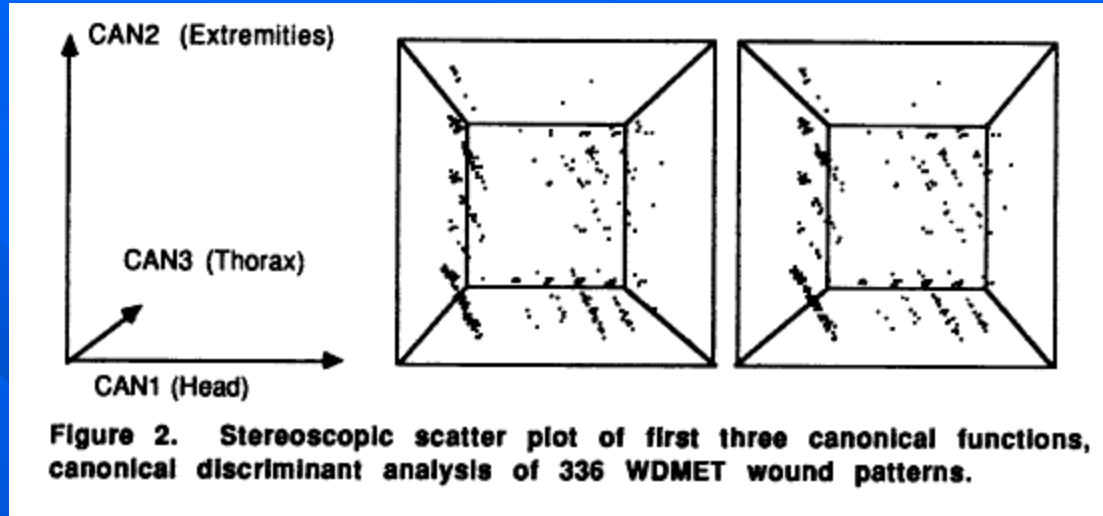


Figure 1.17 Stereo representation of 3 α -contours of a trivariate Normal density. Gently crossing your eyes should allow the 2 frames to fuse in the middle.

Scott (1992)

Applications in Statistics: Then ... (3)

■ Anaglyphs:

- Extensively used within statistics by Carr, Littlefield, and Nicholson (1983-1986): provided construction details for 3-d stereoscopic displays and compared such displays with other visualization techniques for low-dimensional multivariate data sets
- Work by Hering, Symanzik, and von der Weydt (1989-1994): developed computer software to interactively animate anaglyph images for multivariate statistical applications

Applications in Statistics: Then ... (4)

■ Anaglyphs:

PLATE 1 (Banchoff)

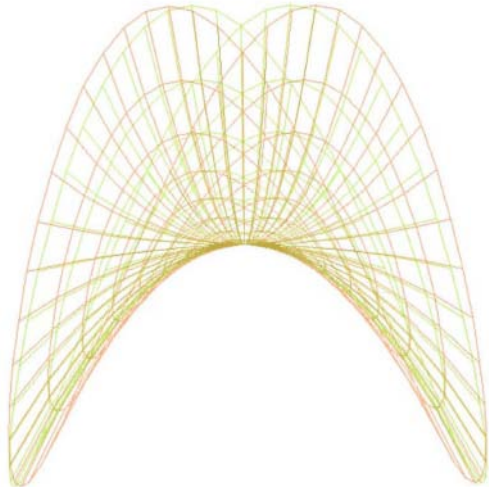


Figure 2

Banchoff (1986)

Symanzik (1992)

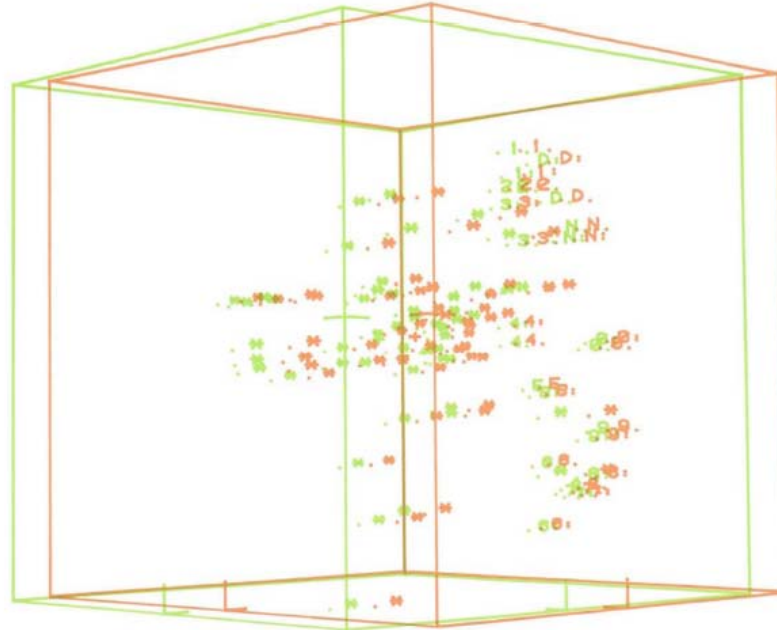


Figure 5 Three-dimensional biplot of American temperature data.

Gabriel &
Odoroff
(1986)

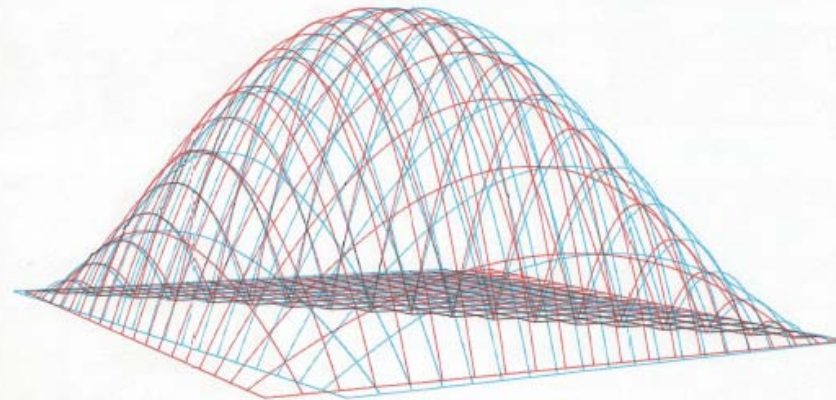


Bild 3: Bivariate Beta-Verteilung

Applications in Statistics: Then ... (5)

■ Anaglyphs:

Carr, Nicholson,
Littlefield & Hall
(1986)

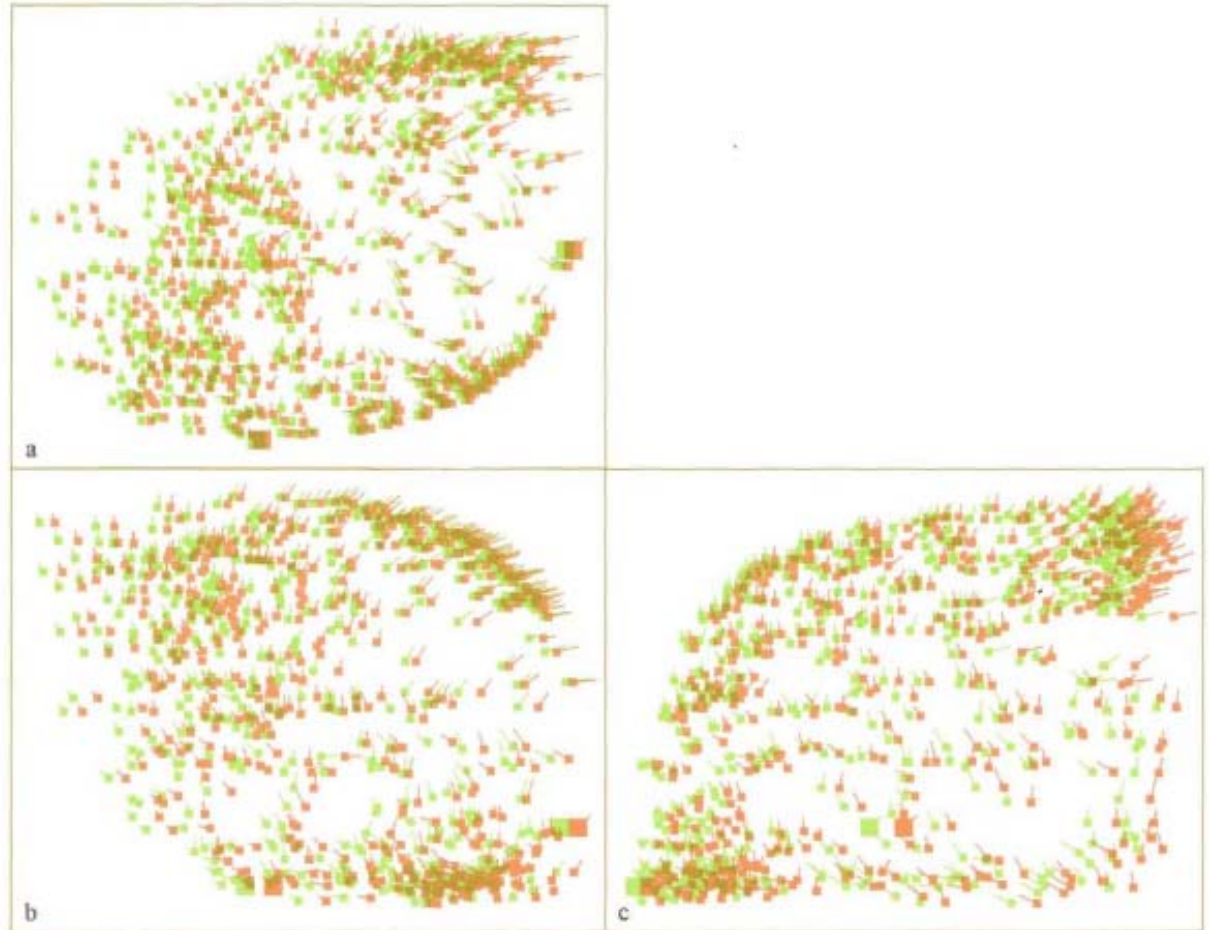


Exhibit 13 Five-dimensional views using stereo ray glyphs. The plot shows three views of five variables, V1, V3, V4, V5, and V7, that partially describe individual replications of a high-energy particle physics scattering experiment (Friedman and Tukey 1974). Exhibit 13b plots V1 as abscissa, V3 as ordinate, and V5 as stereo depth. Exhibit 13a is the view looking into Exhibit 13b from exterior top. Exhibit 13c is the view looking into Exhibit 13b from exterior right. All three views plot V7 as ray orientation and V4 as ray length. Two reference points are plotted with larger ray dots.

Applications in Statistics: ... & Now (1)

Freeviewing of Side-by-Side Images:

– R package lattice (Sarkar 2008):

```
p <-  
  cloud(depth ~ long + lat, quakes, xlim = c(690, 30),  
        pch = ".", cex = 1.5, zoom = 1,  
        xlab = NULL, ylab = NULL, zlab = NULL,  
        par.settings = list(axis.line = list(col = "transparent")),  
        scales = list(draw = FALSE))  
npanel <- 4  
rotz <- seq(-30, 30, length = npanel)  
roty <- c(3, 0)  
  
## Figure 6.3  
update(p[rep(1, 2 * npanel)],  
       layout = c(2, npanel),  
       panel = function(..., screen) {  
         crow <- current.row()  
         ccol <- current.column()  
         panel.cloud(..., screen = list(z = rotz[crow],  
                                         x = -60,  
                                         y = roty[ccol]))  
       })
```

Lattice - Multivariate Data Visualization with R - Figures and Code - Window

http://lmdvr.r-forge.r-project.org/figures/figures.html

File Edit View Favorites Tools Help

Google

Favorites Lattice - Multivariate Data Visualization with R - Figure...

Lattice: Multivariate Data Visualization with R - Figures and Code
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Figure 6.3

Applications in Statistics: ... & Now (2)

■ Anaglyphs:

- R packages `tourr` & `tourrGui` (Cook, Wickham, Huang, Buja, 2011):

```
render(iris[, 1:4], grand_tour(3),  
  display_stereo(blue = rgb(0, 0.91, 0.89),  
    red = rgb(0.98, 0.052, 0)),  
  "pdf", "Iris_Tour_R.pdf", frames = 200)
```

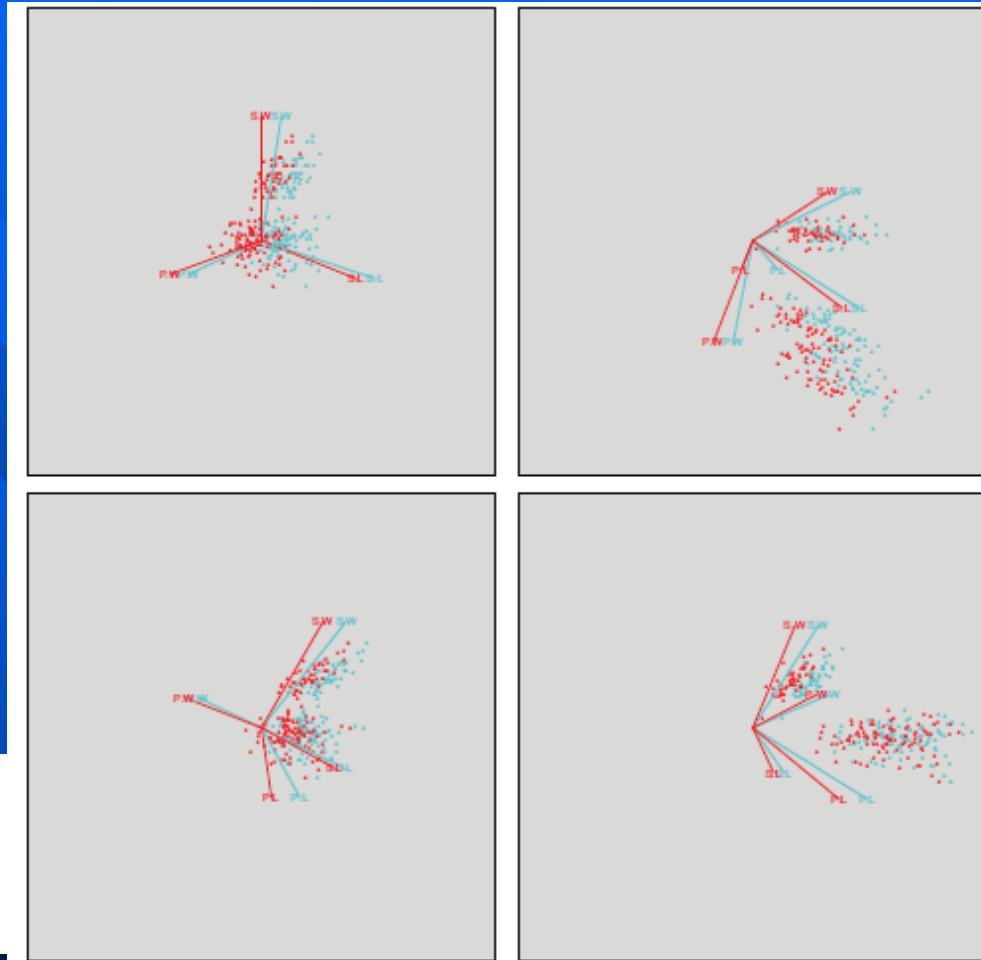
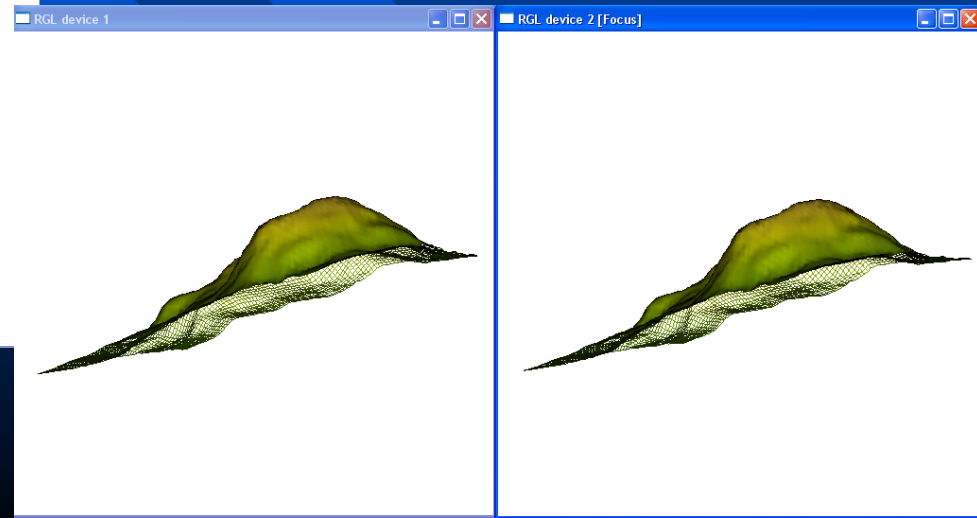
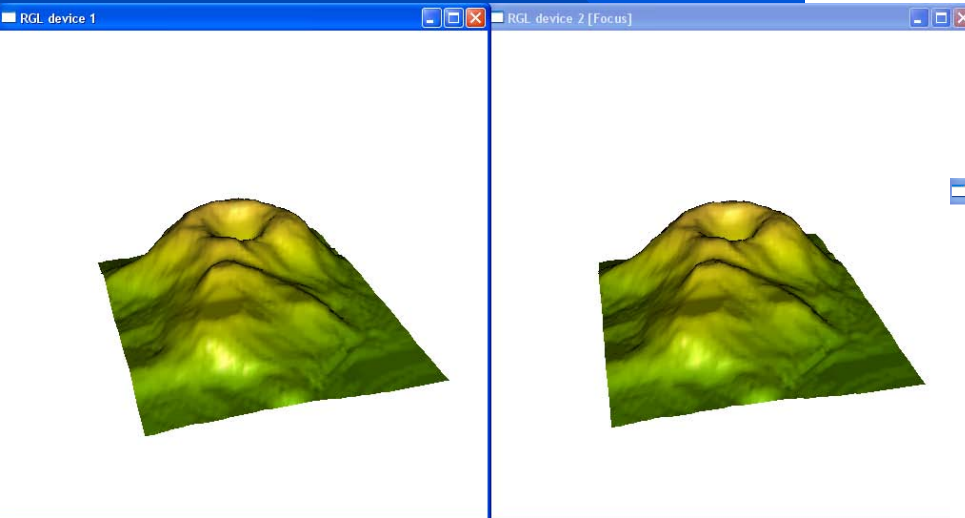
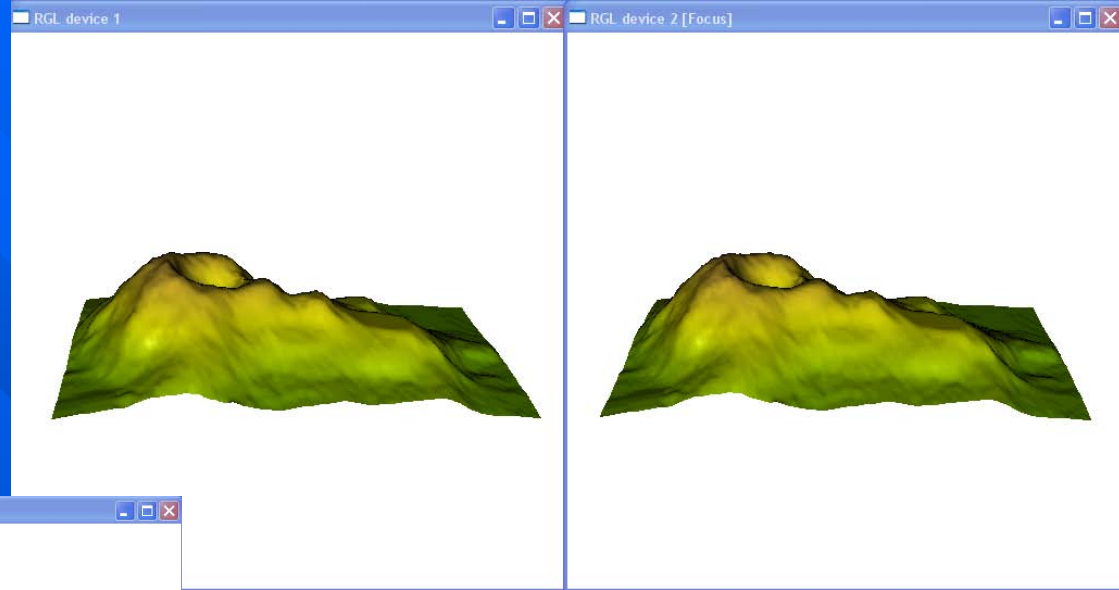


Figure 7: Four different projections of the 3-d grand tour of the Iris data set, shown as red-cyan anaglyphs, were produced via the `tourr` package in R. The abbreviations SL (Sepal Length), SW (Sepal Width), PL (Petal Length), and PW (Petal Width) are used in these plots to denote the four variables of the Iris data set.

Applications in Statistics: ... & Now (3)

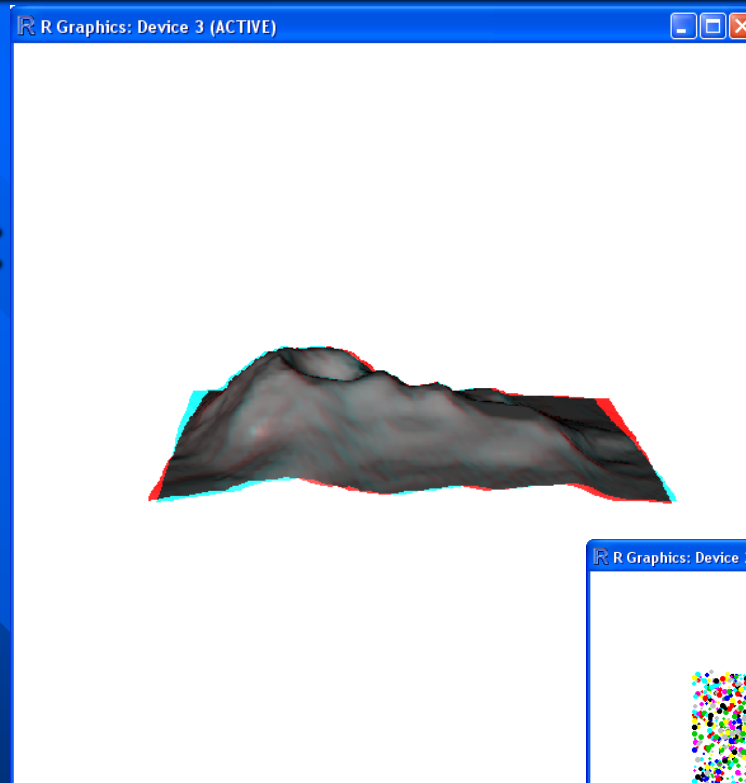
- Freeviewing:
 - R package rgl
(Adler & Murdoch, 2011):



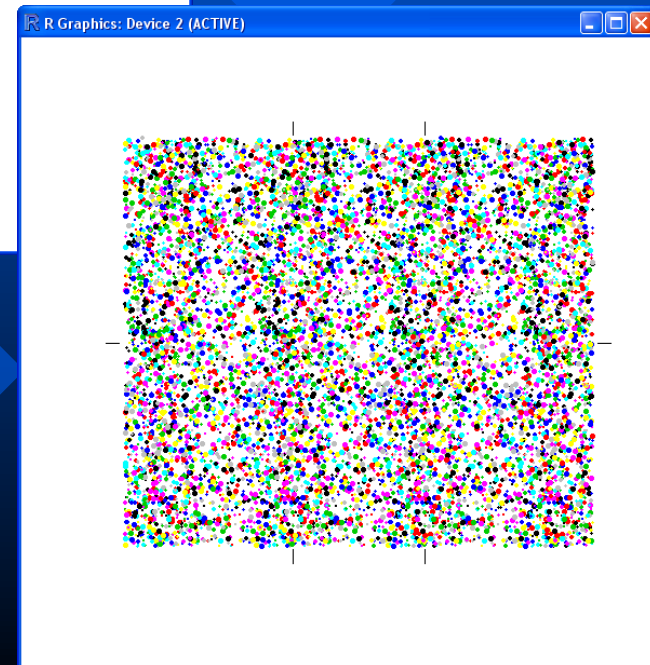
```
> demo("mouseCallbacks")
```

Applications in Statistics: ... & Now (4)

- Anaglyphs & Autostereograms:
 - R package rgl (Adler & Murdoch, 2011):



```
> demo("stereo")
```



Conclusions & Outlook

- 3-D movies (such as Avatar & Toy Story 3) fascinated millions of spectators world-wide
- Leisure books with autostereograms (e.g., Magic Eye series) widely popular since the mid-1990ies
- 3-D TVs and convergence of TVs and computer monitors
- => 3-D stereoscopic graphics (via anaglyphs and others) likely to gain more popularity in statistics in the near future

Main Reference:

Symanzik, J. (2011): 3-d Stereoscopic Plots, Wiley Interdisciplinary Reviews - Computational Statistics (WIREs), Under Review.

Advertisement:

Friday, June 3, 2010

10:15 am – 12:00 noon

Technical Sessions

***Best of Wiley Interdisciplinary Reviews:
Computational Statistics.***

Room 1012

Organizer: Yasmin Said, George Mason University

- 1. *Streaming Data*, William F. Szewczyk, NSA**
- 2. *Record Linkage*, William E. Winkler, Census Bureau**
- 3. *Some Interdisciplinary Topics on: i) Game-Theoretical Solutions for Quantitative Cyber-Risk Estimation/ Management ii) CLOUD Computing Implementations to Compute Operational Risk using Java Programming*, Mehmet Sahinoglu, Auburn University**

Questions ???