

Ch. 13: What are the Chances?

The terms “chance,” “the chances,” or “probability” are used loosely all the time.

Probability was initially developed to solve gambling problems.

For our purposes, we need a more rigorous definition.

_____ is the most common interpretation of probability. The chance of something equals the percentage of the time it is expected to happen if the basic process is repeated over and over again, independently and under the same conditions.

The simplest examples are games of chance, such as those dealing with coins, dice or cards.

Play a Game of Chance

Each person in class flips a coin once:

Number of Heads (H's):

Number of Tails (T's):

Total:

Proportion of H's:

Play again!

What proportion of H's do you expect when we flip the coins 1,000,000 times?

Chance (or probability) makes predictions about long-run behavior (of a coin, die, or whatever).

Chance (or probability) = Proportion in the long run.

Ex: Toss a coin – the chance of H is _____.

Ex: Roll a die – the probability of a “6” is _____.

Facts on Chances

Chances are always between 0% and 100%.

Something impossible has a 0% chance of occurring. Something certain happens 100% of the time. Everything else is in between.

Ex: Chance of 7 in a die roll:

Ex: Chance of a number between 1 and 6 in a die roll:

_____ are the same as chances, but are usually expressed as a decimal or fraction, rather than a percentage.

0% chance	=	probability 0
100% chance	=	probability 1
40% chance	=	probability 0.4
		etc.

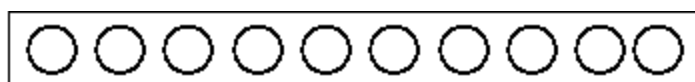
_____ : The chance of something equals 100% minus the chance of the opposite thing.

Ex: If you play a game, and the chance of winning is 45%, the chance of not winning is _____.

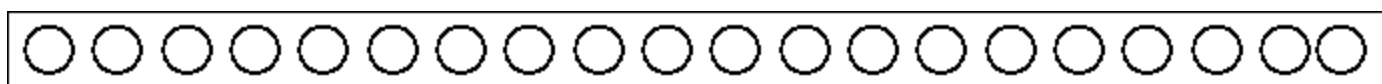
Ex: If we roll a die, we have a probability of $\frac{1}{6}$ to roll a 6. The probability to roll a 1–5 (that is, to *not* roll a 6) is _____.

Drawing Balls from a Box

Ex: Suppose we draw a ball at random out of a box, and win \$1 if the ball is black. We win nothing if the ball is white. If the box contains 4 black balls and 6 white ones (Game A), what is our chance of winning?



Ex: If the box has 5 black and 15 white (Game B), what is our chance of winning?



Which box is better: Game A or Game B?

Ex: And which game would you prefer in this scenario?

Game C:



Game D:



Note:

The probability of drawing a black ball =
$$\frac{\text{number of black balls}}{\text{total number of balls}}$$

Drawing Tickets from a Box

e.g.,

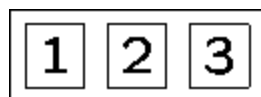
1	2	3	4	5	6
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When drawing more than one ticket, we can draw _____ or _____.

If the draws are made _____ replacement, we draw a ticket, record its number, and then put the ticket back. The chance of getting a certain number doesn't change from draw to draw.

If the draws are made _____ replacement, we draw a ticket, record its number, but then keep the ticket out. The chance of getting a certain number changes in each draw, based on the results of the previous draw.

Ex: Two draws with replacement from

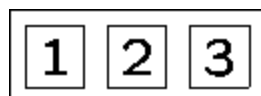


If the first draw is a $\boxed{2}$, then the second ticket is drawn from the box



The chance of drawing a $\boxed{1}$ as the second ticket is:

Ex: Two draws without replacement from



If the first draw is a $\boxed{2}$, then the second ticket is drawn from the box



The chance of drawing a $\boxed{1}$ as the second ticket is:

Playing Cards

Spades: ♠ Clubs: ♣ Diamonds: ♦ Hearts: ♥

Ace

Ace

Ace

Ace

2

2

2

2

3

3

3

3

4

4

4

4

5

5

5

5

6

6

6

6

7

7

7

7

8

8

8

8

9

9

9

9

10

10

10

10

J

J

J

J

Q

Q

Q

Q

K

K

K

K

Ex: Shuffle the pack of 52 cards and deal one.

What is the probability that this card is a ♡?

What is the probability that this card is the Queen of hearts?

What is the probability that this card is a red card?

Conditional Probabilities

Ex: Now consider shuffling the 52 cards and drawing two, placing them face down.

What is the probability that the second card is the Queen of hearts?

Suppose we know the first card is the King of Spades. Now, what is the probability that the second card is the Queen of hearts?

The first probability (or chance) is called _____ – we know nothing.

The second probability (or chance) is called _____ – we know what the first card was.

Independence

Two things are said to be _____ if the chances for the second given the first are the same, no matter how the first turns out. Otherwise, they are *dependent*.

Ex: Are coin tosses independent?

Ex: If we draw with replacement from (1, 1, 2, 2), are the two draws independent?

Ex: What if we draw from the same box *without* replacement?

Ex: If we roll a die twice, are the 2 rolls independent?

Note:

Random draws with replacement are independent.

Random draws without replacement are dependent.

The Multiplication Rule for Independent Events

If two things are independent, the probability that both will happen equals the product of their unconditional probabilities.

Note that this is a special case of the general Multiplication Rule (see page 109).

Ex: Draw 2 cards with replacement from the pack of 52 cards. What is the chance of a ♡, followed by a Queen?

Ex: A couple has 2 children. We assume that the gender of one child is independent from the gender of its siblings.

What is the probability that both children are girls?

And what is the probability that one child is a boy and the other child is a girl?