

Ch. 14: More About Chance

Ex: The roll of a die can give a value of 1, 2, 3, 4, 5, or 6.

The chance of getting a value from {4, 5, 6} is:

The chance of getting a value from {1, 5} is:

We must be careful that all of the ways have equal probability.

Ex: If we roll and sum two dice, we get a number from 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, or 12.

Is the probability we get a 7 equal to $\frac{1}{11} = 9\%$?

There are 21 different two dice combinations:

1-1, 1-2, 1-3, 1-4, 1-5, 1-6,

2-2, 2-3, 2-4, 2-5, 2-6,

3-3, 3-4, 3-5, 3-6,

4-4, 4-5, 4-6,

5-5, 5-6, and

6-6.

Should the chance of a total of 7 be $\frac{3}{21} = 14\%$?

You can figure out chances by listing all the ways an event can happen.

Gambling with 3 Dice

17th century Italian gamblers bet on the sum of 3 dice. They reasoned:

The chance of getting 9 is the same as the chance of getting 10 because there are 6 ways of getting 9 and 6 ways of getting 10:

Ways of getting 9:

1-2-6, 2-3-4,
1-3-5, 2-2-5,
1-4-4, 3-3-3

Ways of getting 10:

1-4-5, 1-3-6,
2-2-6, 2-3-5,
2-4-4, 3-3-4

However, it turned out that 10 came up slightly more often than 9.

They asked Galileo about this, who reasoned as follows: Suppose we have a white die, a grey die, and a black die. We have $6 \times 6 \times 6 = 6^3 = 216$ different combinations.

There is only 1 way to get the numbers 3,3,3:
white = 3, grey = 3, black = 3

There are 3 ways to get the numbers 3,3,4:
white = 3, grey = 3, black = 4
white = 3, grey = 4, black = 3
white = 4, grey = 3, black = 3

There are 6 ways to get the numbers 1,2,6:
white = 1, grey = 2, black = 6
white = 1, grey = 6, black = 2
white = 2, grey = 1, black = 6
white = 2, grey = 6, black = 1
white = 6, grey = 1, black = 2
white = 6, grey = 2, black = 1

Mutually Exclusive Events

Two events are _____ if both cannot happen together.

For each pair of events, are they mutually exclusive?

Ex: The first card is a heart, the first card is a spade?

Ex: The first card is a heart, the second card is a spade?

Ex: The first card is a heart, the first card is an ace?

Ex: The white die is a 1, the black die is a 1?

Ex: The white die is a 1, the sum of white and black dice is 12?

Ex: Two independent events?

What if we want at least one of things which aren't mutually exclusive?

Possibility 1: If *two* events are *not* mutually exclusive, the chance at least one will happen is

$$\text{chance(1st)} + \text{chance(2nd)} - \text{chance(both)}.$$

Ex: When we roll two dice, the chance of getting at least one 6 is:

Ex: When we draw a card from a well-shuffled deck, the chance of a ♡ *or* a Queen is:

Note that this formula only works for *two* events!

Possibility 2: Use the opposites rule. Calculate the chance of the opposite (which will be that *none* of the things occur), and subtract that chance from 100%.

Ex: When we roll two dice, the chance of getting at least one 6 is:

Ex: When we draw a card from a well-shuffled deck, the chance of a ♡ *or* a Queen is: