Ch. 17: The Expected Value and Standard Error

If we repeat a chance process many times, we will get many different results.

In many repetitions of a chance process, the results will vary around the ______________________ (EV), with the amounts they are off being similar in size to the ______________________ (SE).

The expected value of a sum of random draws with replacement from a box equals:

$$\text{EV}_{\text{sum}} = \text{number of draws} \times \text{box average.}$$

__________________________________:

The standard error of a sum of random draws with replacement from a box equals:

$$\text{SE}_{\text{sum}} = \sqrt{\text{number of draws}} \times \text{box SD.}$$

Ex: You play a game in which you roll a die 10 times and get paid the amount shown on the die (each time). The box model is:

What is the amount of money you *expect* to win?

What is the SE for the amount of money you win?

Ex: You play a game in which you roll a die 10 times. Each time a "6" occurs, you win \$ 10, otherwise you lose \$ 1. The box model is:

What is the amount of money you *expect* to win?

What is the SE for the amount of money you win?

Ex: A multiple–choice quiz has 20 questions, each with 4 possible choices. Each correct answer is worth 5 points, and for each incorrect answer you lose 2 points. The box model for your test score (if you guess all the answers) is:

What is the number of points you *expect* to get?

What is the SE for the number of points you get?

Using the Normal Curve

When the number of draws is quite large, we can use the normal curve to calculate chances associated with the sums of draws.

We convert to standard units by using the expected value (instead of the average) and the standard error (instead of the SD).

Standard units say how many SE's we are above or below the EV.

Ex: A multiple–choice quiz has 100 questions, each with 4 possible choices. Each correct answer is worth 1 points, and for each incorrect answer you get 0 points.

Find the chance that someone gets at most 30 points if he or she guesses all the answers.

Find the chance that someone gets at least 60 points if he or she guesses all the answers.

Ex: If a computer repeatedly draws 50 values from

-2	0	1	2	4

and sums them, what percentage of the sums will be between 35 and 65?

Classifying and Counting

If you are interested in the number of times an event occurs, the box has [0] 's and [1] 's.

Ex: Suppose we are rolling a die. If we want the sum of the rolls, we use the box

[1] [2] [3] [4] [5] [6]

On the other hand, if we just want to count 6's, we replace the tickets above with

[0] [0] [0] [0] [0] [1]

In 60 rolls, what is the EV of the number of 6's?

And the SE?

There is a useful shortcut for calculating the average and SD of a box that contains only *two* different numbers:

$$\text{average} = \frac{(\text{smaller} \times \text{how many}) + (\text{bigger} \times \text{how many})}{\text{how many tickets in the box}}$$

$$\text{SD} = (\text{bigger - smaller}) \times \sqrt{\begin{array}{c}\text{fraction}\\ \text{bigger}\end{array} \times \begin{array}{c}\text{fraction}\\ \text{smaller}\end{array}}$$

<u>Ex</u>: You play a game in which you roll a die 10 times. Each time a "6" occurs, you win \$ 10, otherwise you lose \$ 1.

Find the average and SD of the box.

If the numbers in the box are $\boxed{0}$'s and $\boxed{1}$'s, these formula become:

$$\text{average} = \frac{\text{number of } \boxed{1} \text{ 's}}{\text{how many tickets in the box}}$$

$$\text{SD} = \sqrt{\begin{matrix}\text{fraction} \\ \text{of } \boxed{1} \text{ 's}\end{matrix} \times \begin{matrix}\text{fraction} \\ \text{of } \boxed{0} \text{ 's}\end{matrix}}$$

<u>Ex</u>: What is the average and the SD of the box:

$$\boxed{0} \quad \boxed{0} \quad \boxed{0} \quad \boxed{0} \quad \boxed{0} \quad \boxed{1}$$

Ex: 10% of people in a large population are "underweight". If we take a random sample of 200 people from this population, what is the chance that more than 21 will be "underweight"?