Ch. 27: Two-Sample Tests

In this Chapter, we want to look at *two* independent samples from two populations.

Ex: : Suppose we have two boxes (populations): box A and box B.

Box A	Box B
$\text{AVG}_{box} = 110$	$\text{AVG}_{box} = 90$
$\text{SD}_{box} = 60$	$\text{SD}_{box} = 40$

Suppose we draw 400 from box A and 100 from box B and compare the two sample averages:

Sample from Box A	Sample from Box B
$\text{EV}_{avg} =$ ____	$\text{EV}_{avg} =$ ____
$\text{SE}_{sum} =$ ____	$\text{SE}_{sum} =$ ____
$\text{SE}_{avg} =$ ____	$\text{SE}_{avg} =$ ____

We expect the sample average for box A to be _____ higher than the sample average for box B, give or take _____ .

Ooooops – we need another SE!

The SE of the difference of two sample averages is:

$$\text{SE}_{\text{Diff}} = \sqrt{(\text{SE}_{avg_1})^2 + (\text{SE}_{avg_2})^2}.$$

Thus, for our two samples, SE_{Diff} is:

So, we expect the sample averages to differ by 20, give or take _____ .

Similarly,

$$\text{SE}_{\text{Diff}_\%} = \sqrt{(\text{SE}_{\%_1})^2 + (\text{SE}_{\%_2})^2}.$$

Ex: : County A has 45% Republicans. County B has 47% Republicans. We take a random sample of 300 people from County A and 500 from County B.

We expect the percentage of Republicans in the samples to differ by _____, give or take _____.

The two-sample z-test

Box A	Box B
↓	↓
Sample a from box A	Sample b from box B

We want to know if the two population (i.e., box) averages (sums or %) are the same.

Step 1: State hypotheses:

Null hypothesis:
Average for box A equals average for box B.

Alternative hypothesis:
Average for box A is greater / less than average for box B.

Step 2: Calculate test statistic:

$$z = \frac{\text{observed difference - expected difference}}{SE_{\text{Diff}}}.$$

or

$$z = \frac{\text{AVG of Sample a - AVG of Sample b}}{SE_{\text{Diff}}}.$$

Step 3: Obtain P-value:

Same as before - use the normal curve.

<u>Note:</u> The AVG of this normal curve will be zero, the SD will be SE_{Diff}.

Step 4: State conclusions:

Same as before: Based on the P-value, decide whether or not to reject the null hypothesis, and explain your findings in your own words.

Ex: 100 freshman women and 100 freshman men are each given a "Survey of Study Habits and Attitudes." Each individual receives a score from 0 (very poor study habits) to 200 (very good study habits).

The 100 women have an average score of 120, with an SD of 28. The 100 men have an average score of 105, with an SD of 35. Is this reason to believe that the population of freshman women has better study habits than that of freshman men, on average?

Ex: A large university takes a simple random sample of 200 male students, and another simple random sample of 300 females. As it turned out, 107 of the men in the sample used a personal computer on a regular basis, compared to 132 of the sample women. Is there a real difference between the percentages of men and women who use a PC on a regular basis, or is it just a chance variation?

Experiments

This two-sample testing method is commonly used in experiments.

Ex: Two hundred volunteers are used as subjects in a randomized controlled experiment on the effects of regular doses of vitamin C.

The 100 randomly assigned to the treatment (vitamin C) group averaged 2.3 colds, with an SD of 3.1 colds over the period of the experiment. The 100 members of the control group, receiving a placebo, averaged 2.6 colds, with an SD of 2.9 colds. Is there strong reason to believe that regular doses of vitamin C reduce the risk of colds?

When does the two-sample z-test apply?

We can use the two-sample z-test:

- When we have two independent random samples from two populations that we want to compare;

- For randomized controlled experiments – then we *pretend* that the treatment group and the control group are independent random samples.

We can't use the two-sample z-test:

- When our samples are not random;

- When our samples are not independent;

- When we have the whole population – then we know everything about these populations and we can compare them directly – no need for any test!

Ex: An investigator wants to show that first–born children score higher on IQ tests than second–borns. In one school district, he finds 400 two-child families with both children enrolled in elementary school. He gives these children an IQ test and obtains the following results (scores were adjusted for age differences):

– 400 first–borns: AVG = 29; SD = 10.
– 400 second–borns: AVG = 28; SD = 10.

Can he use the two–sample z–test to see whether or not his assumption is true?

Ex: Suppose I teach a class of 200 students, 100 men and 100 women. I give a comprehensive examination, and I find that the average score for the men is 75.4 with an SD of 10.2 and the average score for women is 78.5 with an SD of 10.8 (both sets of scores follow the normal curve closely). Is it appropriate to use the two-sample z–test to decide whether the men and women have different average exam scores?