Ch. 6: Measurement Error

Whenever measurements are made, the answers are not precisely correct.

If we repeat a measurement, we'll probably get a slightly different answer. This is due to __________.

Usually, we don't know where it comes from, but we can estimate how big this error is (Chapter 6) and learn to deal with it (later chapters).

We examine chance error in the context of precision weighing at the National Bureau of Standards (now called the National Institute of Science and Technology, NIST). Below are 5 measurements of a "standard weight" which should be exactly 10 grams:

$$\begin{array}{rl} 9.999 & 591\text{ g} \\ 9.999 & 600\text{ g} \\ 9.999 & 594\text{ g} \\ 9.999 & 601\text{ g} \\ \underbrace{9.999}_{\text{solid}} & \underbrace{598}_{\text{variable}}\text{ g} \end{array}$$

Rewrite these numbers as the amount each measurement falls short of 10 g.

0.000409
0.000400
0.000406
0.000399
0.000402

If we delete the leading 0's and write the differences in micrograms (μg, millionths of a gram), we get:

409
400
406
399
402

These numbers are hard to get a feel for. What does the histogram tell us about the data?

The average (405 μg)?

The SD (6 μg)?

The 100 measurements have a lot of variability, ranging from 375 to 437.

By replicating the measurement many times, we can estimate how large the chance errors are likely to be.

The SD of the measurements is about 6 μg, so the chance error is likely to be around 6 μg.

The SD of a series of measurements is an estimate of the likely size of the chance error in a single measurement.

$$\text{individual measurement} = \underline{\hspace{4cm}} + \underline{\hspace{4cm}}$$

Chance errors can be positive or negative. If we average many measurements, those errors will tend to cancel each other out. The average will still be affected by chance error, but by a lot less than for the individual values.

The average will generally be much closer (have smaller chance error) to the exact value than any individual measurement.

So for our example, we should estimate that the "standard weight" is about 405 μg below 10 g.

Outliers

Out of 100 numbers, how many numbers would you expect to be more than 3 SD's away from the average?

Here, a few measurements are much further away from the average than most. #36 is about 3 SD's from the average; #86 and #94 are about 5 SD's from the average. Such values are called _______________.

These three cases have a big influence on the SD. 86% of the measurements are within 1 SD. If we discard the three outliers, the average changes from 405 to 404, and the SD changes from 6 to 4!

The remaining data follows the normal curve closely.

Should we discard outliers in general?
This depends:

Yes, if ...

No, if ...

Warning: Do not reject valid data simply because it does not fit some theoretical curve.

Ex: As a scientist, you collect 100 measurements to check on a theory you're fond of. All but 2 of the measurements support the theory nicely, but those 2 are very different from the other 98. You're not aware of anything that went wrong when you took those 2 measurements, but you're sure something must have. Should you include those 2 values when you analyze your data?

Bias

In addition to chance error, a measuring process may also contain a consistent error, called __________ or _______________.

individual measurement =

__________ + __________ + __________

If there is no bias, the average of many measurements should be close to the exact value.

If there is a bias, then the average will be close to the exact value + the bias.

Unfortunately, there is no way to tell from a list of measurements alone whether or not the process that collected the numbers is biased!

Ex: What would be the bias in the "standard weight" example with respect to the true weight of 10 grams?