

3. **(6 Points)** The weight of a pair of figure skaters (a male and a female) can be thought of as a new random variable. Define the random variable W as $W = X + Y$. What is the mean μ_W of this new random variable W ?
4. **(6 Points)** Suppose we consider the weights of the male partner and the female partner to be independent. What is the standard deviation σ_W of the random variable W ?
5. **(6 Points)** It does not seem likely that the weights of the male partner and the female partner would be independent. If the correlation ρ between X and Y equals 0.77, what is the standard deviation σ_W of the random variable W ?

Question 2: Statistical Inference (102 Points)

Dichlorodiphenyltrichloroethane (thereafter DDT) is one of the most well-known synthetic pesticides. It is a chemical with a long, unique, and controversial history. First synthesized in 1874, DDT's insecticidal properties were not discovered until 1939, and it was used with great success in the second half of World War II to control malaria and typhus among civilians and troops. The Swiss chemist Paul Hermann Müller was awarded the Nobel Prize in Physiology or Medicine in 1948 "for his discovery of the high efficiency of DDT as a contact poison against several arthropods". After the war, DDT was made available for use as an agricultural insecticide, and soon its production and use skyrocketed.

Unfortunately the pesticide DDT does have a negative side. It causes tremors and convulsions if it is ingested by humans or other mammals. Researchers seek to understand how the convulsions are caused. In a randomized comparative experiment, 5 white rats poisoned with DDT were compared with a control group of 5 unpoisoned rats. Electrical measurements of nerve activity are the main clue to the nature of DDT poisoning. When a nerve is stimulated, its electrical response shows a sharp spike followed by a much smaller second spike. Researchers found that the second spike is larger in rats fed DDT than in normal rats. This observation helps biologists understand how DDT causes tremors. (This example is loosely based on D. L. Shankland, "Involvement of spinal cord and peripheral nerves in DDT-poisoning syndrome in albino rats," *Toxicology and Applied Pharmacology*, 6 (1964), pp. 197–213.) The researchers measured the amplitude of the second spike as a percentage of the first spike when a nerve in the rat's leg was stimulated.

For the poisoned rats the results were:

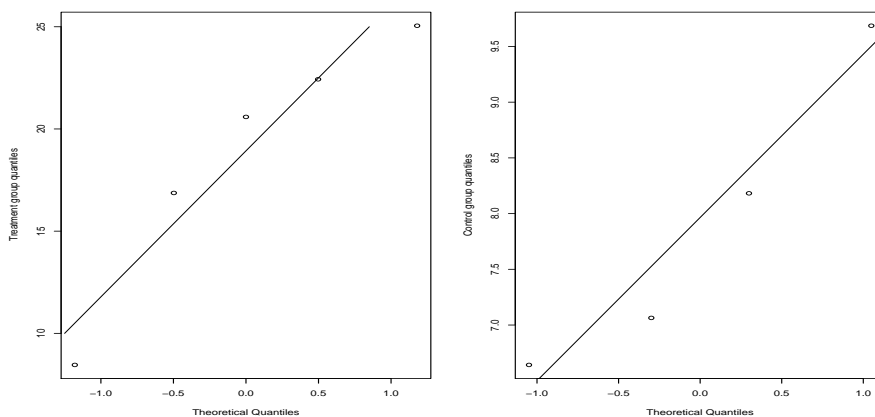
16.869, 25.050, 22.429, 8.456, 20.589.

The control group data were:

9.686, 7.064, 8.182, 6.642.

Note that one rat from the control group escaped before the experiment started.

The researchers created the normal quantile plots to investigate the distributions of data:



Part 1:

By inspecting the data we see that the difference in sample means is quite large, but in such small samples the sample mean is highly variable. A significance test can help confirm that we are seeing a real effect. *Note that the researchers did not conjecture in advance that the size of the second spike would increase or decrease in rats fed DDT.*

Show your work!

1. **(6 Points)** First of all, indicate which test (z -, t -, χ^2 -, one-sided, two-sided, one-sample, two-sample, etc.) you are going to choose, briefly justify your choice, and carefully indicate the crucial assumption(s) that should be satisfied for your test to be valid. Also indicate the ways you would check your assumptions.

5. **(6 Points)** What distribution does your statistic follow. Indicate the name of the distribution and numerical value(s) of the parameters that uniquely describe(s) the distribution of the test statistic. For example, if the test statistic follows the Binomial distribution, you need to specify the numerical value of the sample size and the probability of success, i.e., $X \sim B(n, p)$.
6. **(6 Points)** Determine the P-value.
7. **(6 Points)** If you use the usual 5% significance level, should you reject the null hypothesis? **Yes** / **No** ? Circle your answer and briefly explain why/why not.
8. **(6 Points)** If you use the 1% significance level instead, should you reject the null hypothesis? **Yes** / **No** ? Circle your answer and briefly explain why/why not.
9. **(4 Points)** Give a short summary of your conclusion for the usual 5% significance level, i.e., how would you explain the result to a person who does not know much about statistics?

Part 2:

The researchers recognize that there is likely to be some error in the estimation of the difference in population means. On the basis of known characteristics of the distributions in question, and on our calculation of the standard error of the statistic we are using as an estimate, the researchers would also like to place an interval around the estimator of the difference in population means that specifies the likely range within which the true difference in population means is likely to fall. Therefore they decide to construct a confidence interval. **Show your work!**

1. **(6 Points)** First, what would be a good estimator for the unknown difference in population means? Indicate the proper mathematical symbol(s), and the numerical value(s).
2. **(6 Points)** What is the standard error of the estimator? Indicate the proper mathematical symbol(s), and the numerical value(s).
3. **(6 Points)** Calculate the margin of error for your 95% confidence interval.
4. **(6 Points)** Now, construct a 95% confidence interval for the true difference in population means.

5. **(6 Points)** Can we be certain that the true difference in population parameters falls in this interval? **Yes** / **No** ? Circle your answer.
6. **(6 Points)** What critical value would you use if you wanted 90% confidence instead of 95% confidence? You do not have to calculate this interval — just indicate the value from the table that is needed.
7. **(6 Points)** Would the 90% confidence interval be **(i) wider** or **(ii) narrower** than your 95% confidence interval from 4. (Part 2) above? [Do NOT actually compute the 90% confidence interval — just circle the correct answer.]
8. **(6 Points)** Suppose the alternative hypothesis in 2. (Part 1) is two-sided and $\alpha = 0.05$. Without performing any calculations, just using your confidence interval obtained in 4. (Part 2.) would you reject H_0 ? **Yes** / **No** / **Cannot tell**? Circle your answer. No explanation is needed.

Question 3: Probability and Distributions (48 Points)

During a recent departmental dinner, there were plates with carrot cake and plates with chocolate cake randomly distributed across the tables. We can assume that the probability to obtain a piece of carrot cake is the same as the probability to obtain a piece of chocolate cake. **Show your work!**

1. **(6 Points)** What is the probability that for a table where 6 plates are served, all 6 plates contain carrot cake? Write your answer as a fraction and **not** as a decimal!

2. **(6 Points)** What is the probability that for a table where 6 plates are served, at least 1 plate contains chocolate cake. Write your answer as a fraction and **not** as a decimal!

3. **(6 Points)** There were 15 tables overall that each got 6 plates. Using the proper statistical notation, write down a probability model to describe the underlying distribution that relates to the fact that a table for which 6 plates are served, all 6 plates contain carrot cake (this would be called a success here). Properly use your answer from part 1. above as a part of your answer here. We can assume that the kitchen bakes an extremely large amount of both types of cakes and plates are randomly served at each table.

4. **(6 Points)** Given these 15 tables, what is the expected number of tables that would be served with carrot cake on all 6 plates.

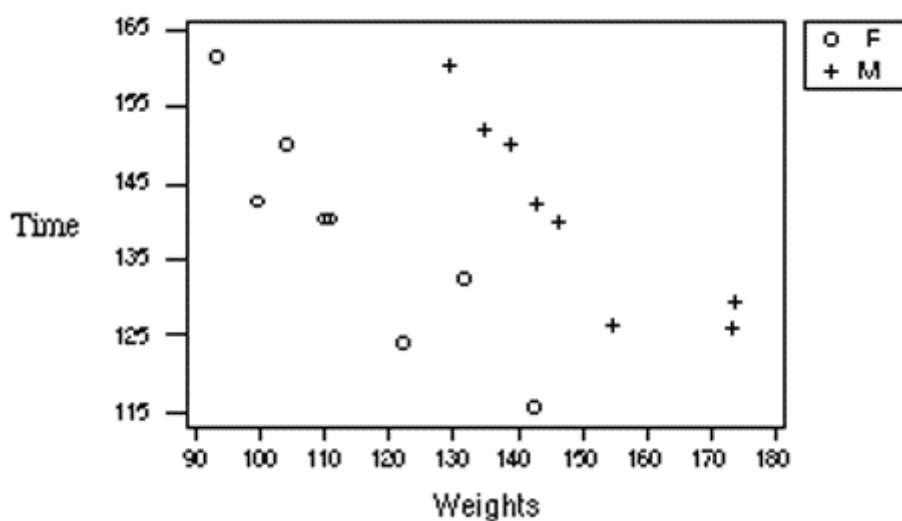
5. **(6 Points)** What is the corresponding standard deviation?
6. **(6 Points)** Can we use a Normal approximation to determine the probability that 7 to 10 of the 15 tables would be served with carrot cake on all 6 plates? **Yes / NO.** Circle the correct answer and explain why you chose this answer.
7. **(6 Points)** Assume that this is not just a departmental dinner, but a university-wide dinner with 1,000 tables on the Quad. The same conditions hold as before, i.e., 6 plates with either carrot cake or chocolate cake are randomly served at each table. Use a Normal approximation to determine the probability that 15 to 20 of the 1,000 tables would be served with carrot cake on all 6 plates. For the purpose of this question part, we believe that the necessary requirements hold.
8. **(6 Points)** Now, do the same calculation as in the previous part, but use a Normal approximation with **continuity correction**.

Question 4: Multiple Choice Questions (120 Points)

Mark your answer for each multiple choice question in the table below. There is only one correct answer for each question. Each correct answer is worth 4 points.

Question	(a)	(b)	(c)	(d)	Question	(a)	(b)	(c)	(d)
1	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	16	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
2	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	17	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
3	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	18	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
4	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	19	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
5	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	20	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
6	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	21	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
7	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	22	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
8	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	23	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
9	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	24	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
10	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	25	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
11	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	26	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
12	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	27	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
13	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	28	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
14	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	29	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
15	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	30	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

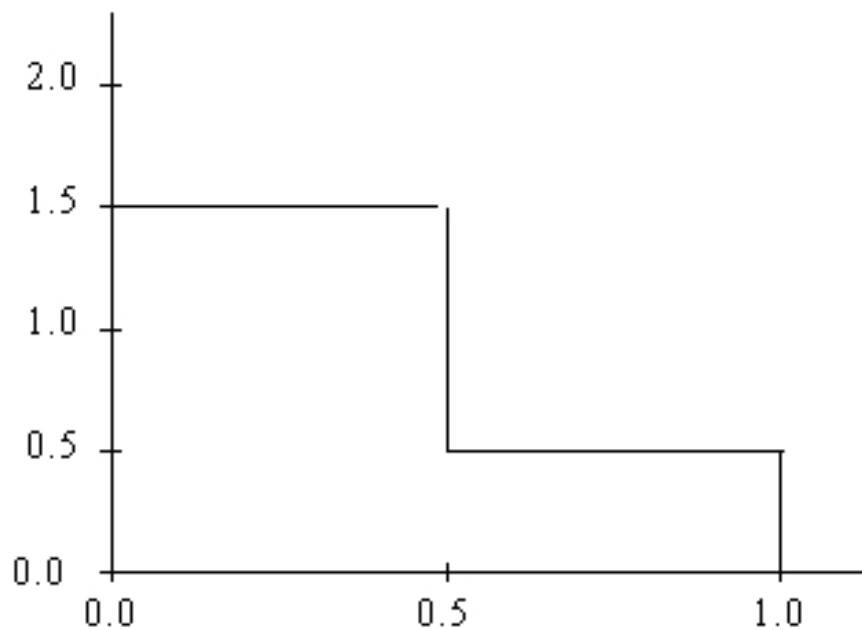
- On a chilly spring afternoon, 10 lab sections of a statistics class all have full attendance. The 10 lab sections each have the same number of students enrolled in it. A class evaluation is about to be administered to some of the students. It has been decided to first randomly select 3 of the 10 lab sections and then give the evaluation to a simple random sample of one-fourth of the students in those sections. What sampling technique is being used?
 - Convenience sampling.
 - Simple random sampling.
 - Stratified random sampling.
 - Multistage sampling.
- The data in the scatterplot below are an individual's weight and the time it takes (in seconds) on a treadmill to raise his or her pulse rate to 140 beats per minute. The o 's correspond to females and the $+$'s to males.



Based on the scatterplot we conclude

- there is a positive correlation r between gender and weight, since men tend to weigh more.
- there is a negative correlation between time and weight for males and for females.
- both (a) and (b) are correct.
- both (a) and (b) are incorrect.

3. There are two statistics classes. The first has 250 students and the second has 200 students. In the first class the students are instructed to toss a coin 20 times and record the value of \hat{p} , the proportion of heads. The instructor then makes a histogram of the 250 values of \hat{p} obtained. In the second class the students are instructed to toss a coin 40 times and record the value of \hat{p} , the proportion of heads. The instructor then makes a histogram of the 200 values of \hat{p} obtained. The histogram of \hat{p} values for the **first** class should be
- (a) more biased since it is based on a smaller number of tosses.
 - (b) more variable since it is based on a smaller number of tosses.
 - (c) less variable since it is based on a larger number of students.
 - (d) roughly equally variable as the smaller number of tosses is cancelled out by the larger number of students.
4. Suppose X is a continuous random variable taking values between 0 and 1 and having a probability distribution described by the following density curve:



The probability that X takes a value between 0 and $3/4$ is

- (a) 0.875.
- (b) 0.75.
- (c) $3/8$.
- (d) 0.25.

5. Does taking garlic tablets twice a day provide significant health benefits? To investigate this issue, a researcher conducted a study of 50 adult subjects who took garlic tablets twice a day for a period of six months. At the end of the study, 100 variables related to the health of the subjects were measured on each subject and the means compared to known means for these variables in the population of all adults. Four of these variables were significantly better (in the sense of statistical significance) at the 5% level for the group taking the garlic tablets as compared to the population as a whole, and one variable was significantly better at the 1% level for the group taking the garlic tablets as compared to the population as a whole. It would be correct to conclude
- (a) there is good statistical evidence that taking garlic tablets twice a day provides some health benefits.
 - (b) there is good statistical evidence that taking garlic tablets twice a day provides benefits for the variable that was significant at the 1% level. We should be somewhat cautious about making claims for the variables that were significant at the 5% level.
 - (c) there is good statistical evidence that taking garlic tablets twice a day hurts the subject.
 - (d) None of the above.
6. As part of a promotion for a new type of cracker, free trial samples are offered to shoppers in a local supermarket. The probability that a shopper will buy a packet of crackers after tasting the free sample is 0.3. Different shoppers can be regarded as independent trials. If \hat{p} is the proportion of the next 100 shoppers that buy a packet of the crackers after tasting a free sample, then \hat{p} has approximately a Normal distribution with mean $\mu = 0.3$ and
- (a) standard deviation $\sigma = 21$.
 - (b) standard deviation $\sigma = 0.21$.
 - (c) standard deviation $\sigma = 0.0458$.
 - (d) standard deviation $\sigma = 0.0021$.
7. Suppose we are testing the null hypothesis $H_0 : \mu = 50$ and the alternative $H_a : \mu \neq 50$ for a normal population with $\sigma = 6$. The 95% confidence interval for the mean is (51.3, 54.7). Then
- (a) the P-value for the test is greater than 0.10.
 - (b) the P-value for the test is between than 0.05 and 0.10.
 - (c) the P-value for the test is less than 0.05.
 - (d) the P-value for the test could be anything. It can't be determined without knowing the sample size.

8. The one sample t statistic from a sample of $n = 19$ observations for the two-sided test of $H_0 : \mu = 6$ vs. $H_a : \mu \neq 6$ has the value $t = 1.93$. Based on this information
- (a) we would reject the null hypothesis at $\alpha = 0.10$.
 - (b) $0.025 < \text{P-value} < 0.05$.
 - (c) both (a) and (b) are correct.
 - (d) both (a) and (b) are incorrect.
9. The scores on a university examination are normally distributed with a mean of 62 and a standard deviation of 8. If the top 2% of students are given A's, what is the lowest mark that a student can have and still be awarded an A?
- (a) 87.63.
 - (b) 80.64.
 - (c) 78.40.
 - (d) 45.60.
10. You receive a fax with six temperature measurements taken in January (in degrees Fahrenheit)
- 8, -6, -2, 0, -4 and x ,
- where x is some number that is too blurry to read. Without knowing what x is, the median
- (a) could be any number between -2 and 8.
 - (b) must be between -4 and 0.
 - (c) is -2.
 - (d) is 0.
11. In the table below we examine the relationship between final grade and the reported hours per week each student said they studied for the course.

	A	B	C	D	F	Total
< 5	2	3	7	4	5	21
5 – 10	3	5	5	4	6	23
> 10	6	5	3	4	2	20
Total	11	13	15	12	13	64

The probability that a student earned a B if he studied less than 5 hours per week is

- (a) $3/64$.
- (b) $3/13$.
- (c) $21/64$.
- (d) $1/7$.

12. Suppose you are going to roll a die 120 times and record \hat{p} , the proportion of times that a 2 or a 3 or a 4 or a 5 or a 6 is showing. The sampling distribution of \hat{p} should be centered about
- (a) $5/6$.
 - (b) $1/2$.
 - (c) $1/3$.
 - (d) $4/6$.
13. In a particular game, a ball is randomly chosen from a box that contains 6 red balls, 1 green ball, and 3 blue balls. If a red ball is selected you win \$20, if a green ball is selected you win \$40, and if a blue ball is selected you win nothing. Let X be the amount that you win. The expected value of X is
- (a) nothing (\$0).
 - (b) \$15.
 - (c) \$16.
 - (d) \$20.
14. On a certain airline, the chance the early flight from Atlanta to Chicago is full is 0.7. The chance the late flight is full is 0.6. The chance both flights are full is 0.42. Are the two flights being full independent events?
- (a) Yes.
 - (b) No.
 - (c) We do not have enough information.
 - (d) There must be something wrong with this question as the probabilities sum up to 1.72 — and this is impossible!
15. Correlation is a measure of the direction and strength of the linear (straight-line) association between two quantitative variables. The analysis of data from a study found that the scatterplot between two variables, x and y , appeared to show a straight-line relationship and the correlation was calculated to be -0.84 . This tells us that
- (a) there is little reason to believe that the two variables have a linear association relationship.
 - (b) all of the data values for the two variables lie on a straight line.
 - (c) there is a strong linear relationship between the two variables with larger values of x tending to be associated with larger values of the y variable.
 - (d) there is a strong linear relationship between x and y with smaller x values tending to be associated with larger values of the y variable.

16. Incomes in a certain town are strongly right skewed with mean \$100,000 and standard deviation \$80,000. A random sample of 5 households is taken. What is the probability the average of the sample is more than \$240,000?
- (a) 0.1831.
 - (b) 0.3875
 - (c) 0.9750.
 - (d) Cannot say.
17. For moderate samples sizes, confidence intervals based on the t -distribution (i.e., “ t -intervals”) are _____ confidence intervals based on the Normal distribution (i.e., “ z -intervals”) based on the same data set.
- (a) narrower than
 - (b) the same as
 - (c) wider than
 - (d) exactly twice as wide as
18. For a χ^2 -test that is based on a table that consists of 4 rows and 4 columns, the value of the chi-square statistic is 17.422. The corresponding P-value is
- (a) less than 0.0005.
 - (b) between 0.0005 and 0.025.
 - (c) between 0.025 and 0.05.
 - (d) greater than 0.05.
19. A table has 4 rows and 5 columns. The degrees of freedom for a χ^2 test are
- (a) 9.
 - (b) 10.
 - (c) 12.
 - (d) 20.
20. The lifetime of a 2-volt non-rechargeable battery in constant use has a normal distribution with a mean of 516 hours and a standard deviation of 20 hours. 90% of all batteries have a lifetime less than
- (a) 490.40 hours.
 - (b) 517.28 hours.
 - (c) 536.00 hours.
 - (d) 541.60 hours.

21. An experimenter is convinced that there is such a phenomenon as extrasensory perception. To demonstrate this a randomly selected group of subjects were given a test designed to show their ability. Following the collection of the data a test of significance showed that the results were not significant at the 0.05 level of significance. Undaunted, the experimenter repeated the experiment several times and finally obtained an experimental result with a P-value of 0.03. What can properly be said about this result?
- (a) The result should be published because it is an important finding in this subject area.
 - (b) The P-value of 0.03 proves that the experimenter's faith was justified.
 - (c) The significant P-value proves that there is such a phenomenon as ESP.
 - (d) The experimenter is guilty of searching for significance and the result should be discounted.

Use the following to answer questions 22 and 23:

The distribution of GPA scores is known to be left-skewed. At a large university, an English professor is interested in learning about the average GPA score of the English majors and minors. A simple random sample of 75 junior and senior English majors and minors results in an average GPA score of 2.97. Assume that the distribution of GPA scores for all English majors and minors at this university is also left-skewed with a standard deviation of 0.62.

22. Calculate a 95% confidence interval for the mean GPA of the junior and senior English majors and minors.
- (a) (2.786, 3.154).
 - (b) (2.830, 3.110).
 - (c) (2.852, 3.088).
 - (d) (2.954, 2.986).
23. Determine which of the following statements is true.
- (a) The probability that the *population mean* falls between the bounds of the confidence interval calculated in the previous question equals 0.95.
 - (b) If many samples of 75 English students were taken and many 95% confidence intervals calculated, only 5% of the time the *sample mean* would not fall in one of those confidence intervals.
 - (c) If many samples of 75 English students were taken and many 95% confidence intervals calculated, only 5% of the time the *population mean* would not fall in one of those confidence intervals.
 - (d) If many samples of 75 English students were taken and many 95% confidence intervals calculated, only 5% of the time the *sample mean* would not fall between the bounds of the confidence interval calculated in the previous question.

Use the following to answer questions 24, 25, and 26:

In statistics, we usually refer to x_1 as the first observation, x_2 as the second observation, etc., and x_n as the final observation when we write down our observations in the order they were obtained (where n represents the total number of observations).

Often, we prefer to work with data that are sorted from smallest to largest, e.g., when calculating the median, we need the data to be sorted. Obviously, we can simply reorder any given list of numbers. However, we often use the notation $x_{(1)}$ to refer to the smallest observation, $x_{(2)}$ to refer to the 2nd smallest observation, etc., and $x_{(n)}$ to refer to the largest observation.

24. For $x_1 = 2, x_2 = 5, x_3 = -3, x_4 = -5, x_5 = -1, x_6 = 10$, and $n = 6$, the sum

$$\sum_{i=2}^{n-1} x_i =$$

equals

- (a) -4.
 - (b) -1.
 - (c) 3.
 - (d) 8.
25. For $x_1 = 2, x_2 = 5, x_3 = -3, x_4 = -5, x_5 = -1, x_6 = 10$, and $n = 6$, the sum

$$\sum_{i=2}^{n-1} x_{(i)} =$$

equals

- (a) -4.
 - (b) -1.
 - (c) 3.
 - (d) 8.
26. For $x_1 = 2, x_2 = 5, x_3 = -3, x_4 = -5, x_5 = -1, x_6 = 10$, and $n = 6$, the sum

$$\sum_{i=1}^n x_i =$$

equals

- (a) 2.
- (b) 8.
- (c) 10.
- (d) 12.

27. The mathematical expression $4!$ (read 4 factorial) numerically results in
- (a) 1.
 - (b) 4.
 - (c) 12.
 - (d) 24.
28. The mathematical expression $0!$ (read 0 factorial) numerically results in
- (a) -1 .
 - (b) 0.
 - (c) 1.
 - (d) nothing — this is an invalid calculation.
29. The mathematical expression $\binom{5}{3}$ (read 5 choose 3) numerically results in
- (a) 1.
 - (b) 5.
 - (c) 10.
 - (d) 120.
30. The mathematical expression $\binom{5}{5}$ (read 5 choose 5) numerically results in
- (a) 1.
 - (b) 5.
 - (c) 10.
 - (d) 120.

TABLES

You can remove the following 2 sheets of tables from the exam. No need to turn in these tables once they have been removed.

Table entry for z is the area under the standard normal curve to the left of z .

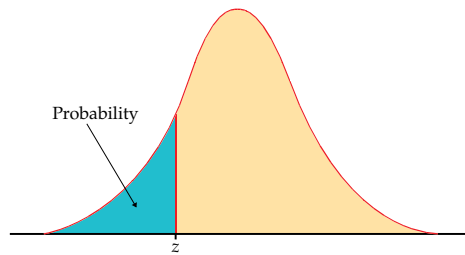


TABLE A										
Standard normal probabilities										
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

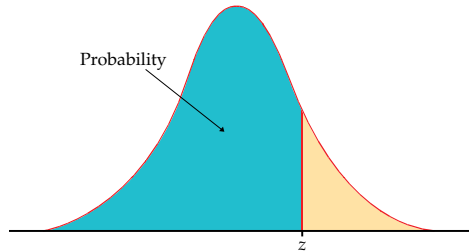


Table entry for z is the area under the standard normal curve to the left of z .

TABLE A										
Standard normal probabilities (continued)										
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table entry for p and C is the critical value t^* with probability p lying to its right and probability C lying between $-t^*$ and t^* .

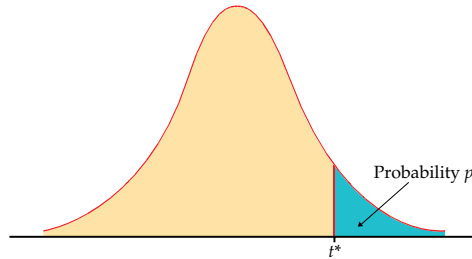
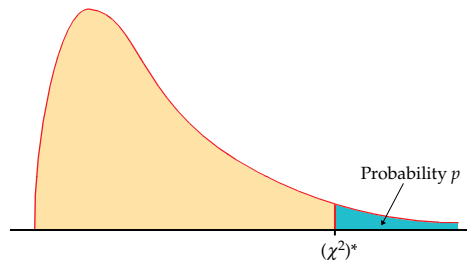


TABLE D												
t distribution critical values												
df	Upper-tail probability p											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

Table entry for p is the critical value $(\chi^2)^*$ with probability p lying to its right.



df	Tail probability p											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2