# Statistics 2000, Section 001, Final (300 Points)

Wednesday, May 4, 2011

## Part I: Text Answers

**Your Name:** _____

**Question 1:** Statistical Inference (**68 Points**)

Eight people volunteered to be part of an experiment. All 8 people were Caucasian, between the ages of 25 and 35, and were supplied with nice clothes. 4 of the people were male and 4 were female. The question of interest in this experiment was whether females receive faster service at restaurants than males. Each of the 4 male participants was randomly assigned to a restaurant, and each of the 4 females was randomly assigned to one of these **same (!)** 4 restaurants. One Friday night, all 8 people went out to eat, each one alone. The male and female assigned to the same restaurant would arrive within 5 minutes of each other, with the order determined by flipping a coin (male first or female first). Each person then ordered a similar drink and a similar meal. The time (in minutes) until the food arrived at the table was recorded. They are shown below:

| Restaurant | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Male | 22 | 14 | 16 | 26 |
| Female | 25 | 12 | 13 | 21 |

It is reasonable to believe that the differences in serving times at the same restaurant roughly follow a normal distribution.
**Show your work!**

1. (**8 Points**) We want to determine whether women are served **faster** than men. State the two hypotheses. Be careful what goes into the null hypothesis and what goes into the alternative. Use the proper mathematical notation and symbols.

2. (**6 Points**) Do we have to conduct a **matched pairs t–test** or a **2–sample t–test**? Circle the correct test and briefly justify your answer.

3. (**9 Points**) Calculate the proper mean(s) and standard deviation(s). No need to show your work for this part — just write down the final results.

4. (**10 Points**) Calculate the test statistic.

5. (**6 Points**) What are the appropriate degrees of freedom for this test?

6. (**8 Points**) Determine the P–value.

7. (**7 Points**) If you use the usual 5% significance level, should you reject the null hypothesis? **Yes / No ?** Circle your answer and briefly explain why/why not.

8. (**7 Points**) If you use the 20% significance level instead, should you reject the null hypothesis? **Yes / No ?** Circle your answer and briefly explain why/why not.

9. (**7 Points**) Give a short summary of your conclusion for the usual 5% significance level, i.e., how would you explain the result to a person who does not know much about statistics?

**Question 2:** Regression (**37 Points**)

Data were obtained from the A&W Web site for the Total Fat in grams and the Protein content in grams for various items on their menu. Some summary statistics are also provided:

| Item | Total Fat (grams) | Protein (grams) |
|---|---|---|
| Kid's Cheeseburger | 24 | 23 |
| Kid's Hamburger | 22 | 21 |
| Original Bacon Cheeseburger | 33 | 27 |
| Original Bacon Double Cheeseburger | 48 | 45 |
| Original Double Cheeseburger | 42 | 40 |
| Papa Burger | 42 | 41 |

| | Total Fat | Protein |
|---|---|---|
| Mean | 35.167 | 32.833 |
| Standard Deviation | 10.591 | 10.362 |
| Correlation | r = 0.983 | |

The scatterplot (not reproduced here) shows that there is indeed a linear relationship between the two variables.

**Work with 3 decimal digits (as above) and show your work!**

1. (**16 Points**) Find the regression equation for predicting the Protein content from the Total Fat content.

2. (**5 Points**) Using your regression equation, estimate the Protein content for a menu item with a Total Fat content of 40 grams.

3. (**8 Points**) The "Kid's Cheeseburger" contains 24 grams Total Fat and 23 grams Protein. Using your regression equation, calculate the corresponding residual (i.e., error). Be careful with the sign!

4. (**8 Points**) The recently introduced "Triple Meat Mega Fat Burger" (not yet listed on the menu above) contains 80 grams Total Fat. Is it safe to use your regression equation from 1. above to predict the Protein content for this burger? **Yes / No. Circle your answer and provide an explanation.**

**Question 3:** General Statistical Concepts (**30 Points**)

Yves Rocher, a leading producer of cosmetics, includes the information below in the package of its brand new anti–wrinkle night creme:


1. (**18 Points**) Assume that you are invited to a press conference where this new product is presented. You are allowed to ask **THREE** different **statistical** questions that apparently have not been answered on the package information above. You are not allowed to ask non–statistical questions such as the retail price or stores where this new product will be sold. Be precise in formulating your questions, i.e., use proper statistical terms in your questions! You can assume that the head of the Yves Rocher Statistics Department will be at the press conference and can answer your questions.

   1)

   2)

   3)

2. (**12 Points**) Actually, the two numbers from the package information above (i.e., 48% and 20 women) represent the only statistical information revealed by Yves Rocher for this new product. Can you speculate why none of your questions has been answered in the full package information? We can assume that a well–trained statistician such as the head of the Yves Rocher Statistics Department should have anticipated most of your questions before they were even asked... Provide **TWO** likely reasons why Yves Rocher may have chosen to not reveal additional statistical information. Briefly explain each reason in 1 (or at most 2) sentences.

   1)

   2)

**Question 4:** Probability and Expectations (**45 Points**)

A child is looking for the preferred kind of pralines as a Mothers' Day gift for its mother. There are 5 warehouses in town. Unknown to the child, only 2 of these 5 warehouses carry this particular kind of pralines. The child visits warehouse after warehouse until it finds the preferred kind of pralines. The child **does not** go to the same warehouse twice! Once the child has found the preferred kind of pralines, it does not go to any additional warehouse.
Let $X$ be the random variable that represents the number of warehouses the child has to visit until it finds the preferred kind of pralines. $X$ can only take values 1, 2, 3, and 4.
**Show your work!**

1. (**8 Points**) Sketch a (probability) tree diagram that summarizes the information from the text above. Label the paths and nodes of your tree in meaningful ways.

2. (**12 Points**) Use your probability tree above to read off (or calculate) the following probabilities:

   $P(X = 1) =$

   $P(X = 2) =$

   $P(X = 3) =$

   $P(X = 4) =$

3. (**4 Points**) We have discussed several probability rules in Chapter 4. Verify that the two most basic rules hold in your probability assignments above. Hint: Rule 1 is related to the probability of any event $A$ and Rule 2 is related to the probability of the sample space $S$.

4. (**8 Points**) What is the expected (average) number of warehouses the child has to visit until it finds the preferred kind of pralines?

5. (**7 Points**) Given that the child **did not** find the preferred kind of pralines in warehouse 1, what is the probability that the child will find the preferred kind of pralines in warehouse 2?

6. (**6 Points**) Does $X$ have a $Bin(5, 2/5)$ distribution? **Yes / No. Circle your answer and provide an explanation.**

## Statistics 2000, Section 001, Final (300 Points)

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## Part II: Multiple Choice Questions

**Your Name:** _____

**Question 5:** Multiple Choice Questions **(120 Points)**

**Mark your answer for each multiple choice question in the table below. There is only one correct answer for each question.** The first 25 questions have four choices, the last 5 questions have five choices. Each correct answer is worth 4 points.

| Question | (a) | (b) | (c) | (d) | Question | (a) | (b) | (c) | (d) | (e) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | ◯ | ◯ | ◯ | ◯ | 16 | ◯ | ◯ | ◯ | ◯ | |
| 2 | ◯ | ◯ | ◯ | ◯ | 17 | ◯ | ◯ | ◯ | ◯ | |
| 3 | ◯ | ◯ | ◯ | ◯ | 18 | ◯ | ◯ | ◯ | ◯ | |
| 4 | ◯ | ◯ | ◯ | ◯ | 19 | ◯ | ◯ | ◯ | ◯ | |
| 5 | ◯ | ◯ | ◯ | ◯ | 20 | ◯ | ◯ | ◯ | ◯ | |
| 6 | ◯ | ◯ | ◯ | ◯ | 21 | ◯ | ◯ | ◯ | ◯ | |
| 7 | ◯ | ◯ | ◯ | ◯ | 22 | ◯ | ◯ | ◯ | ◯ | |
| 8 | ◯ | ◯ | ◯ | ◯ | 23 | ◯ | ◯ | ◯ | ◯ | |
| 9 | ◯ | ◯ | ◯ | ◯ | 24 | ◯ | ◯ | ◯ | ◯ | |
| 10 | ◯ | ◯ | ◯ | ◯ | 25 | ◯ | ◯ | ◯ | ◯ | |
| 11 | ◯ | ◯ | ◯ | ◯ | 26 | ◯ | ◯ | ◯ | ◯ | ◯ |
| 12 | ◯ | ◯ | ◯ | ◯ | 27 | ◯ | ◯ | ◯ | ◯ | ◯ |
| 13 | ◯ | ◯ | ◯ | ◯ | 28 | ◯ | ◯ | ◯ | ◯ | ◯ |
| 14 | ◯ | ◯ | ◯ | ◯ | 29 | ◯ | ◯ | ◯ | ◯ | ◯ |
| 15 | ◯ | ◯ | ◯ | ◯ | 30 | ◯ | ◯ | ◯ | ◯ | ◯ |

1. You are thinking of using a t–procedure to construct a 95% confidence interval for the mean of a population. You suspect the distribution of the population is not normal and may be strongly skewed. Which of the following statements is correct?

   (a) You should not use the t–procedure since the population does not have a normal distribution.

   (b) You may use the t–procedure provided your sample size is large, say at least 40 or 50.

   (c) You may use the t–procedure provided your sample size is at least 15.

   (d) You may use the t–procedure since it is robust to non–normality.

2. A (fair) coin is tossed multiple times. Which of the following outcome sequences is more likely? Note that H represents that the coin lands on heads and T represents that the coin lands on tails.

   (a) H, H, H, H.

   (b) H, T, T, H, T.

   (c) T, T, T, T, T.

   (d) They are all equally likely.

3. A call–in poll conducted by *USA Today* concluded that Americans love Donald Trump. This conclusion was based on data collected from 7800 calls made by *USA Today* readers. What sampling technique is being used?

   (a) Simple random sampling.

   (b) Stratified random sampling.

   (c) Volunteer sampling.

   (d) Convenience sampling.

4. An amateur gardener decides to change the variety of tomatoes for this year to see if the yield improves. He put in six plants the previous year and six plants this year using the same part of the garden. The average yield per plant was 11.3 pounds per plant in the previous year and 14.5 pounds per plant using the new variety. What is this an example of?

   (a) An experiment.

   (b) An observational study, not an experiment.

   (c) The elimination of all confounding variables by design, because the gardener used the same part of the garden in both years.

   (d) A multistage design, because two years were involved.

5. For small samples, t intervals are _____ z intervals based on the same data set.

   (a) narrower than

   (b) the same as

   (c) wider than

   (d) Can't be determined without doing the actual calculations.

6. As the degrees of freedom become larger, the difference between the t and z distributions becomes _____.

   (a) narrower

   (b) stays the same

   (c) wider

   (d) Can't be determined without knowing the actual sample size.

7. When a particular penny is held on its edge and spun, the probability that heads are face up when the coin comes to rest is 4/9. If the coin is spun 4 times, the probability that the coin will come up heads exactly twice is (assume trials are independent)

   (a) 4/9.

   (b) 16/81.

   (c) 0.366.

   (d) 0.061.

8. Suppose that scores on the Math SAT exam follow a normal distribution with mean 500 and standard deviation 100. Two students that have taken the exam are selected at random. What is the probability that the sum of their scores exceeds 1200?

   (a) 0.1587.

   (b) $0.1587^2$.

   (c) 0.0793.

   (d) $0.9207^2$.

**Use the following to answer questions 9, 10, and 11:**

We are interested in comparing the proportions of males and females who think earning a large salary is very important to them. I surveyed 200 of each gender and recorded their answers to the question as "yes" or "no". Results are summarized in the table below:

| | Yes | No | Total |
|---|---|---|---|
| Males | 150 | 50 | 200 |
| Females | 110 | 90 | 200 |
| Total | 260 | 140 | 400 |

9. The expected number of males who think earning a large salary is very important is

   (a) 40.

   (b) 100.

   (c) 130.

   (d) 150.

10. The degrees of freedom for a $\chi^2$ test based on this table are

    (a) 1.

    (b) 4.

    (c) 2 and 2.

    (d) 399.

11. The null hypothesis for a $\chi^2$ test based on this table is

    (a) $H_0$: There is no association.

    (b) $H_0$: "Gender" and "Salary Importance" are dependent.

    (c) $H_0$: The population has a higher proportion of men than women.

    (d) $H_0$: The population has a higher proportion of people who think earning a large salary is very important to them than people who think otherwise.

12. As part of a promotion for a new type of cracker, free samples are offered to shoppers in a local supermarket. The probability that a shopper will buy a packet of crackers after tasting the free sample is 0.200. Different shoppers can be regarded as independent trials. Let $\hat{p}$ be the proportion of the next $n$ shoppers that buy a packet of the crackers after tasting a free sample. How large should $n$ be so that the standard deviation of $\hat{p}$ is no more than 0.01?

    (a) 4.

    (b) 16.

    (c) 400.

    (d) 1600.

13. The regression line to predict average exam grade from hours of study is

$$y = 15 + 5.6 \cdot x.$$

    The slope of the regression line indicates

    (a) for any student, an extra hour of study increases the grade 5.6 points.

    (b) on average, an extra hour of study will increase the grade 5.6 points.

    (c) an extra hour of study will increase the grade 15 points.

    (d) The relationship is meaningless as there are many confounding factors.

14. Assuming the population is large, which sample size will give the smallest standard deviation to a statistic (such as the sample mean)?

    (a) $n = 100$.

    (b) $n = 500$.

    (c) $n = 1000$.

    (d) There is no difference, no matter which value is selected for $n$.

15. Assuming the population is large, which sample size will give the least bias to a statistic?

    (a) $n = 100$.

    (b) $n = 500$.

    (c) $n = 1000$.

    (d) We don't know.

16. I have computed an Analysis of Variance (ANOVA) for 4 groups with 15 observations per group. The degrees of freedom for the F test are

    (a) 3 and 14.

    (b) 3 and 56.

    (c) 4 and 56.

    (d) 4 and 60.

17. I have computed an Analysis of Variance (ANOVA) F test for 4 groups with 15 observations per group and obtained a p–value of 0.19. This means

    (a) We can assume that all the groups have the same mean.

    (b) We can assume that all the group means are different.

    (c) We can assume that some of the group means differ from the others.

    (d) The result is biased as we only have 15 observations in each of the 4 groups.

18. I have 4 groups for which I want to perform an ANOVA. They have standard deviations $s_1 = 2.5, s_2 = 3.4, s_3 = 4.5, s_4 = 2.3$ and the plots of the data indicate all samples are approximately normal with no outliers. Is the ANOVA appropriate?

    (a) No.

    (b) Yes — because the data are approximately normal distributed with no outliers.

    (c) Yes — because the largest sample standard deviation is no more than twice as big as the smallest sample standard deviation.

    (d) Yes — because both (b) and (c) are fulfilled (just one of these is not enough).

19. The three basic principles of statistical design of experiments are

    (a) pairing, comparison, confounding.

    (b) comparison, randomization, repetition.

    (c) blocking, blinding, bias avoidance.

    (d) randomization, control, pairing.

20. It has been claimed that women live longer than men; however, men tend to be older than their wives. Ages of 16 deceased husbands and wives from England were obtained via a simple random sample of death records. These data should be analyzed with a

    (a) Two-sample t-test.

    (b) Paired samples t-test

    (c) Two-sample z-test.

    (d) $\chi^2$ test.

**Use the following to answer questions 21 and 22:**

In order to assess the opinion of students at the University of Minnesota about campus snow removal, a reporter for the student newspaper interviews the first 12 students he meets who are willing to express their opinions.

21. What type of sampling method was used to collect the data?

    (a) A simple random sample.

    (b) A stratified random sample.

    (c) Voluntary response.

    (d) A census.

22. What set of students make up the sample?

    (a) All those students favoring prompt snow removal.

    (b) All students at universities receiving substantial snow.

    (c) The 12 students interviewed.

    (d) All students at the University of Minnesota.

**Use the following to answer questions 23, 24, and 25:**

In statistics, we usually refer to $x_1$ as the first observation, $x_2$ as the second observation, etc., and $x_n$ as the final observation when we write down our observations in the order they were obtained (where $n$ represents the total number of observations).

Often, we prefer to work with data that are sorted from smallest to largest, e.g., when calculating the median, we need the data to be sorted. Obviously, we can simply reorder any given list of numbers. However, we often use the notation $x_{(1)}$ to refer to the smallest observation, $x_{(2)}$ to refer to the 2nd smallest observation, etc., and $x_{(n)}$ to refer to the largest observation.

23. For $x_1 = 8, x_2 = 5, x_3 = -5, x_4 = -10, x_5 = 10$, and $n = 5$, the expression

$$x_{\left(\frac{n+1}{2}\right)} =$$

equals

(a) $-10$.
(b) 5.
(c) 8.
(d) 10.

24. For $x_1 = 8, x_2 = 5, x_3 = -5, x_4 = -10, x_5 = 10$, and $n = 5$, the sum

$$\sum_{i=2}^{n-1} x_i =$$

equals

(a) $-10$.
(b) 5.
(c) 8.
(d) 10.

25. For $x_1 = 8, x_2 = 5, x_3 = -5, x_4 = -10, x_5 = 10$, and $n = 5$, the sum

$$\sum_{i=1}^{3} x_{(i)} =$$

equals

(a) $-10$.
(b) 5.
(c) 8.
(d) 10.

26. Ignoring twins and other multiple births, assume babies born at a hospital are independent events with the probability that a baby is a boy and the probability that a baby is a girl both equal to 0.5. Define events $A = \{\text{the next two babies are boys}\}$ and $B = \{\text{at least one of the next two babies is a boy}\}$. What do we know about events $A$ and $B$?

    (a) They are disjoint.

    (b) They are complements.

    (c) They are independent.

    (d) $P(A) = P(B) = 1/2$.

    (e) None of the above.

27. Let $X$ be a random variable with mean $\mu_X = 25$ and $\sigma_X = 6$ and let $Y$ be a random variable with mean $\mu_Y = 30$ and $\sigma_Y = 4$. It is known that $X$ and $Y$ are independent random variables. Suppose the random variable $Y$ is subtracted from $X$ so that a new random variable $U$ is created, i.e., $U = X - Y$. What is the standard deviation of $U$?

    (a) $\sigma_U = 2$.

    (b) $\sigma_U = 20$.

    (c) $\sigma_U = 7.2$.

    (d) $\sigma_U = 52$.

    (e) None of the above.

28. Which of the following statements about a scatterplot is TRUE?

    (a) It is always necessary to identify one of the two variables as the explanatory variable and the other as the response variable.

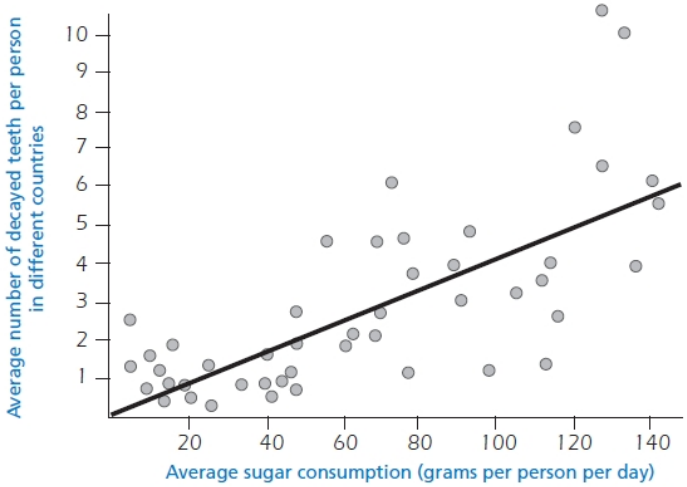
    (b) On a scatterplot we look for overall patterns showing the form, direction, and the shape of the relationship.

    (c) Because a scatterplot requires the values of two quantitative variables it is never possible to add one or more categorical variables to the graph.

    (d) Both (a) and (b) are true statements.

    (e) None of the above.

29. The following graph shows the consumption of sugar and the amount of caries in different countries. Each country is represented by a dot in the graph.




Which one of the following statements is supported *by the data given in the graph*?

(a) In some countries, people brush their teeth more frequently than in other countries.

(b) The more sugar people eat, the more likely they are to get caries.

(c) In recent years, the rate of caries has increased in many countries.

(d) In recent years, the consumption of sugar has increased in many countries.

(e) None of the above.

30. Let $X$ be a binomial random variable with distribution $B(10, 0.6)$. What is the probability that $X$ equals 8?

(a) $0.6^2 \cdot 0.4^8$.

(b) $\frac{10!}{8!} \cdot 0.6^8 \cdot 0.4^2$.

(c) $45 \cdot 0.6^2 \cdot 0.4^8$.

(d) $45 \cdot 0.6^8 \cdot 0.4^2$.

(e) None of the above.

# TABLES

**You can remove the following 2 sheets of tables from the exam. No need to turn in these tables once they have been removed.**



Table entry for $z$ is the area under the standard normal curve to the left of $z$.

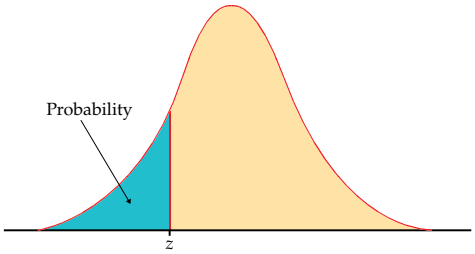
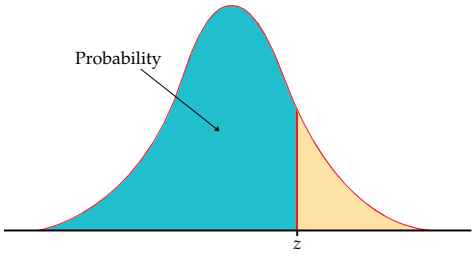



**TABLE A**

Standard normal probabilities

| $z$ | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| −3.4 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0002 |
| −3.3 | .0005 | .0005 | .0005 | .0004 | .0004 | .0004 | .0004 | .0004 | .0004 | .0003 |
| −3.2 | .0007 | .0007 | .0006 | .0006 | .0006 | .0006 | .0006 | .0005 | .0005 | .0005 |
| −3.1 | .0010 | .0009 | .0009 | .0009 | .0008 | .0008 | .0008 | .0008 | .0007 | .0007 |
| −3.0 | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |
| −2.9 | .0019 | .0018 | .0018 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| −2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| −2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| −2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| −2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0048 |
| −2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| −2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| −2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| −2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| −2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| −1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| −1.8 | .0359 | .0351 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| −1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| −1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| −1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| −1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0721 | .0708 | .0694 | .0681 |
| −1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| −1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| −1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| −1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| −0.9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| −0.8 | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| −0.7 | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| −0.6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| −0.5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| −0.4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| −0.3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3483 |
| −0.2 | .4207 | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| −0.1 | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| 0.0 | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |

Table entry for $z$ is the area under the standard normal curve to the left of $z$.

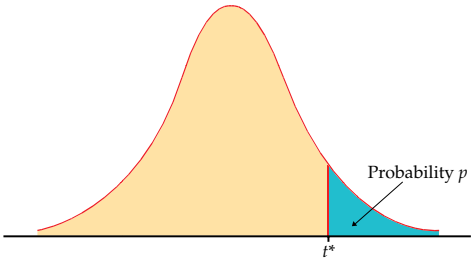



## TABLE A

### Standard normal probabilities (continued)

| $z$ | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| 0.1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| 0.2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| 0.3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| 0.4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| 0.5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| 0.6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| 0.7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| 0.8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| 0.9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |

Table entry for $p$ and $C$ is the critical value $t^*$ with probability $p$ lying to its right and probability $C$ lying between $-t^*$ and $t^*$.

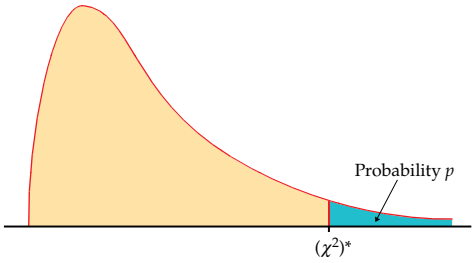



## TABLE D

### *t* distribution critical values

| | Upper-tail probability $p$ | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| df | .25 | .20 | .15 | .10 | .05 | .025 | .02 | .01 | .005 | .0025 | .001 | .0005 |
| 1 | 1.000 | 1.376 | 1.963 | 3.078 | 6.314 | 12.71 | 15.89 | 31.82 | 63.66 | 127.3 | 318.3 | 636.6 |
| 2 | 0.816 | 1.061 | 1.386 | 1.886 | 2.920 | 4.303 | 4.849 | 6.965 | 9.925 | 14.09 | 22.33 | 31.60 |
| 3 | 0.765 | 0.978 | 1.250 | 1.638 | 2.353 | 3.182 | 3.482 | 4.541 | 5.841 | 7.453 | 10.21 | 12.92 |
| 4 | 0.741 | 0.941 | 1.190 | 1.533 | 2.132 | 2.776 | 2.999 | 3.747 | 4.604 | 5.598 | 7.173 | 8.610 |
| 5 | 0.727 | 0.920 | 1.156 | 1.476 | 2.015 | 2.571 | 2.757 | 3.365 | 4.032 | 4.773 | 5.893 | 6.869 |
| 6 | 0.718 | 0.906 | 1.134 | 1.440 | 1.943 | 2.447 | 2.612 | 3.143 | 3.707 | 4.317 | 5.208 | 5.959 |
| 7 | 0.711 | 0.896 | 1.119 | 1.415 | 1.895 | 2.365 | 2.517 | 2.998 | 3.499 | 4.029 | 4.785 | 5.408 |
| 8 | 0.706 | 0.889 | 1.108 | 1.397 | 1.860 | 2.306 | 2.449 | 2.896 | 3.355 | 3.833 | 4.501 | 5.041 |
| 9 | 0.703 | 0.883 | 1.100 | 1.383 | 1.833 | 2.262 | 2.398 | 2.821 | 3.250 | 3.690 | 4.297 | 4.781 |
| 10 | 0.700 | 0.879 | 1.093 | 1.372 | 1.812 | 2.228 | 2.359 | 2.764 | 3.169 | 3.581 | 4.144 | 4.587 |
| 11 | 0.697 | 0.876 | 1.088 | 1.363 | 1.796 | 2.201 | 2.328 | 2.718 | 3.106 | 3.497 | 4.025 | 4.437 |
| 12 | 0.695 | 0.873 | 1.083 | 1.356 | 1.782 | 2.179 | 2.303 | 2.681 | 3.055 | 3.428 | 3.930 | 4.318 |
| 13 | 0.694 | 0.870 | 1.079 | 1.350 | 1.771 | 2.160 | 2.282 | 2.650 | 3.012 | 3.372 | 3.852 | 4.221 |
| 14 | 0.692 | 0.868 | 1.076 | 1.345 | 1.761 | 2.145 | 2.264 | 2.624 | 2.977 | 3.326 | 3.787 | 4.140 |
| 15 | 0.691 | 0.866 | 1.074 | 1.341 | 1.753 | 2.131 | 2.249 | 2.602 | 2.947 | 3.286 | 3.733 | 4.073 |
| 16 | 0.690 | 0.865 | 1.071 | 1.337 | 1.746 | 2.120 | 2.235 | 2.583 | 2.921 | 3.252 | 3.686 | 4.015 |
| 17 | 0.689 | 0.863 | 1.069 | 1.333 | 1.740 | 2.110 | 2.224 | 2.567 | 2.898 | 3.222 | 3.646 | 3.965 |
| 18 | 0.688 | 0.862 | 1.067 | 1.330 | 1.734 | 2.101 | 2.214 | 2.552 | 2.878 | 3.197 | 3.611 | 3.922 |
| 19 | 0.688 | 0.861 | 1.066 | 1.328 | 1.729 | 2.093 | 2.205 | 2.539 | 2.861 | 3.174 | 3.579 | 3.883 |
| 20 | 0.687 | 0.860 | 1.064 | 1.325 | 1.725 | 2.086 | 2.197 | 2.528 | 2.845 | 3.153 | 3.552 | 3.850 |
| 21 | 0.686 | 0.859 | 1.063 | 1.323 | 1.721 | 2.080 | 2.189 | 2.518 | 2.831 | 3.135 | 3.527 | 3.819 |
| 22 | 0.686 | 0.858 | 1.061 | 1.321 | 1.717 | 2.074 | 2.183 | 2.508 | 2.819 | 3.119 | 3.505 | 3.792 |
| 23 | 0.685 | 0.858 | 1.060 | 1.319 | 1.714 | 2.069 | 2.177 | 2.500 | 2.807 | 3.104 | 3.485 | 3.768 |
| 24 | 0.685 | 0.857 | 1.059 | 1.318 | 1.711 | 2.064 | 2.172 | 2.492 | 2.797 | 3.091 | 3.467 | 3.745 |
| 25 | 0.684 | 0.856 | 1.058 | 1.316 | 1.708 | 2.060 | 2.167 | 2.485 | 2.787 | 3.078 | 3.450 | 3.725 |
| 26 | 0.684 | 0.856 | 1.058 | 1.315 | 1.706 | 2.056 | 2.162 | 2.479 | 2.779 | 3.067 | 3.435 | 3.707 |
| 27 | 0.684 | 0.855 | 1.057 | 1.314 | 1.703 | 2.052 | 2.158 | 2.473 | 2.771 | 3.057 | 3.421 | 3.690 |
| 28 | 0.683 | 0.855 | 1.056 | 1.313 | 1.701 | 2.048 | 2.154 | 2.467 | 2.763 | 3.047 | 3.408 | 3.674 |
| 29 | 0.683 | 0.854 | 1.055 | 1.311 | 1.699 | 2.045 | 2.150 | 2.462 | 2.756 | 3.038 | 3.396 | 3.659 |
| 30 | 0.683 | 0.854 | 1.055 | 1.310 | 1.697 | 2.042 | 2.147 | 2.457 | 2.750 | 3.030 | 3.385 | 3.646 |
| 40 | 0.681 | 0.851 | 1.050 | 1.303 | 1.684 | 2.021 | 2.123 | 2.423 | 2.704 | 2.971 | 3.307 | 3.551 |
| 50 | 0.679 | 0.849 | 1.047 | 1.299 | 1.676 | 2.009 | 2.109 | 2.403 | 2.678 | 2.937 | 3.261 | 3.496 |
| 60 | 0.679 | 0.848 | 1.045 | 1.296 | 1.671 | 2.000 | 2.099 | 2.390 | 2.660 | 2.915 | 3.232 | 3.460 |
| 80 | 0.678 | 0.846 | 1.043 | 1.292 | 1.664 | 1.990 | 2.088 | 2.374 | 2.639 | 2.887 | 3.195 | 3.416 |
| 100 | 0.677 | 0.845 | 1.042 | 1.290 | 1.660 | 1.984 | 2.081 | 2.364 | 2.626 | 2.871 | 3.174 | 3.390 |
| 1000 | 0.675 | 0.842 | 1.037 | 1.282 | 1.646 | 1.962 | 2.056 | 2.330 | 2.581 | 2.813 | 3.098 | 3.300 |
| $z^*$ | 0.674 | 0.841 | 1.036 | 1.282 | 1.645 | 1.960 | 2.054 | 2.326 | 2.576 | 2.807 | 3.091 | 3.291 |
| | 50% | 60% | 70% | 80% | 90% | 95% | 96% | 98% | 99% | 99.5% | 99.8% | 99.9% |
| | Confidence level $C$ | | | | | | | | | | | |

Table entry for $p$ is the critical value $(\chi^2)^*$ with probability $p$ lying to its right.




## TABLE F

### $\chi^2$ distribution critical values

| | Tail probability $p$ | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| df | .25 | .20 | .15 | .10 | .05 | .025 | .02 | .01 | .005 | .0025 | .001 | .0005 |
| 1 | 1.32 | 1.64 | 2.07 | 2.71 | 3.84 | 5.02 | 5.41 | 6.63 | 7.88 | 9.14 | 10.83 | 12.12 |
| 2 | 2.77 | 3.22 | 3.79 | 4.61 | 5.99 | 7.38 | 7.82 | 9.21 | 10.60 | 11.98 | 13.82 | 15.20 |
| 3 | 4.11 | 4.64 | 5.32 | 6.25 | 7.81 | 9.35 | 9.84 | 11.34 | 12.84 | 14.32 | 16.27 | 17.73 |
| 4 | 5.39 | 5.99 | 6.74 | 7.78 | 9.49 | 11.14 | 11.67 | 13.28 | 14.86 | 16.42 | 18.47 | 20.00 |
| 5 | 6.63 | 7.29 | 8.12 | 9.24 | 11.07 | 12.83 | 13.39 | 15.09 | 16.75 | 18.39 | 20.51 | 22.11 |
| 6 | 7.84 | 8.56 | 9.45 | 10.64 | 12.59 | 14.45 | 15.03 | 16.81 | 18.55 | 20.25 | 22.46 | 24.10 |
| 7 | 9.04 | 9.80 | 10.75 | 12.02 | 14.07 | 16.01 | 16.62 | 18.48 | 20.28 | 22.04 | 24.32 | 26.02 |
| 8 | 10.22 | 11.03 | 12.03 | 13.36 | 15.51 | 17.53 | 18.17 | 20.09 | 21.95 | 23.77 | 26.12 | 27.87 |
| 9 | 11.39 | 12.24 | 13.29 | 14.68 | 16.92 | 19.02 | 19.68 | 21.67 | 23.59 | 25.46 | 27.88 | 29.67 |
| 10 | 12.55 | 13.44 | 14.53 | 15.99 | 18.31 | 20.48 | 21.16 | 23.21 | 25.19 | 27.11 | 29.59 | 31.42 |
| 11 | 13.70 | 14.63 | 15.77 | 17.28 | 19.68 | 21.92 | 22.62 | 24.72 | 26.76 | 28.73 | 31.26 | 33.14 |
| 12 | 14.85 | 15.81 | 16.99 | 18.55 | 21.03 | 23.34 | 24.05 | 26.22 | 28.30 | 30.32 | 32.91 | 34.82 |
| 13 | 15.98 | 16.98 | 18.20 | 19.81 | 22.36 | 24.74 | 25.47 | 27.69 | 29.82 | 31.88 | 34.53 | 36.48 |
| 14 | 17.12 | 18.15 | 19.41 | 21.06 | 23.68 | 26.12 | 26.87 | 29.14 | 31.32 | 33.43 | 36.12 | 38.11 |
| 15 | 18.25 | 19.31 | 20.60 | 22.31 | 25.00 | 27.49 | 28.26 | 30.58 | 32.80 | 34.95 | 37.70 | 39.72 |
| 16 | 19.37 | 20.47 | 21.79 | 23.54 | 26.30 | 28.85 | 29.63 | 32.00 | 34.27 | 36.46 | 39.25 | 41.31 |
| 17 | 20.49 | 21.61 | 22.98 | 24.77 | 27.59 | 30.19 | 31.00 | 33.41 | 35.72 | 37.95 | 40.79 | 42.88 |
| 18 | 21.60 | 22.76 | 24.16 | 25.99 | 28.87 | 31.53 | 32.35 | 34.81 | 37.16 | 39.42 | 42.31 | 44.43 |
| 19 | 22.72 | 23.90 | 25.33 | 27.20 | 30.14 | 32.85 | 33.69 | 36.19 | 38.58 | 40.88 | 43.82 | 45.97 |
| 20 | 23.83 | 25.04 | 26.50 | 28.41 | 31.41 | 34.17 | 35.02 | 37.57 | 40.00 | 42.34 | 45.31 | 47.50 |
| 21 | 24.93 | 26.17 | 27.66 | 29.62 | 32.67 | 35.48 | 36.34 | 38.93 | 41.40 | 43.78 | 46.80 | 49.01 |
| 22 | 26.04 | 27.30 | 28.82 | 30.81 | 33.92 | 36.78 | 37.66 | 40.29 | 42.80 | 45.20 | 48.27 | 50.51 |
| 23 | 27.14 | 28.43 | 29.98 | 32.01 | 35.17 | 38.08 | 38.97 | 41.64 | 44.18 | 46.62 | 49.73 | 52.00 |
| 24 | 28.24 | 29.55 | 31.13 | 33.20 | 36.42 | 39.36 | 40.27 | 42.98 | 45.56 | 48.03 | 51.18 | 53.48 |
| 25 | 29.34 | 30.68 | 32.28 | 34.38 | 37.65 | 40.65 | 41.57 | 44.31 | 46.93 | 49.44 | 52.62 | 54.95 |
| 26 | 30.43 | 31.79 | 33.43 | 35.56 | 38.89 | 41.92 | 42.86 | 45.64 | 48.29 | 50.83 | 54.05 | 56.41 |
| 27 | 31.53 | 32.91 | 34.57 | 36.74 | 40.11 | 43.19 | 44.14 | 46.96 | 49.64 | 52.22 | 55.48 | 57.86 |
| 28 | 32.62 | 34.03 | 35.71 | 37.92 | 41.34 | 44.46 | 45.42 | 48.28 | 50.99 | 53.59 | 56.89 | 59.30 |
| 29 | 33.71 | 35.14 | 36.85 | 39.09 | 42.56 | 45.72 | 46.69 | 49.59 | 52.34 | 54.97 | 58.30 | 60.73 |
| 30 | 34.80 | 36.25 | 37.99 | 40.26 | 43.77 | 46.98 | 47.96 | 50.89 | 53.67 | 56.33 | 59.70 | 62.16 |
| 40 | 45.62 | 47.27 | 49.24 | 51.81 | 55.76 | 59.34 | 60.44 | 63.69 | 66.77 | 69.70 | 73.40 | 76.09 |
| 50 | 56.33 | 58.16 | 60.35 | 63.17 | 67.50 | 71.42 | 72.61 | 76.15 | 79.49 | 82.66 | 86.66 | 89.56 |
| 60 | 66.98 | 68.97 | 71.34 | 74.40 | 79.08 | 83.30 | 84.58 | 88.38 | 91.95 | 95.34 | 99.61 | 102.7 |
| 80 | 88.13 | 90.41 | 93.11 | 96.58 | 101.9 | 106.6 | 108.1 | 112.3 | 116.3 | 120.1 | 124.8 | 128.3 |
| 100 | 109.1 | 111.7 | 114.7 | 118.5 | 124.3 | 129.6 | 131.1 | 135.8 | 140.2 | 144.3 | 149.4 | 153.2 |